Introduction to Three-phase Circuits

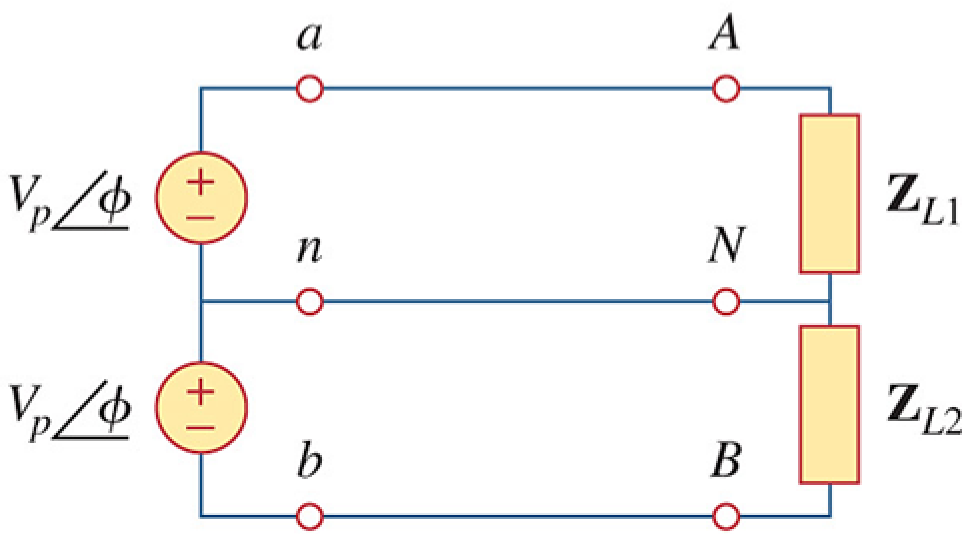
Balanced 3-phase systems

Unbalanced 3-phase systems

Introduction to 3-phase systems

Single-phase two-wire system:

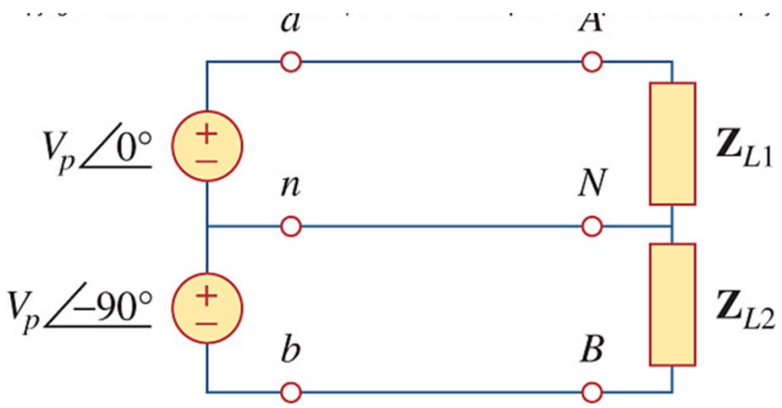
- Single source connected to a load using two-wire system



Single-phase three-wire system:

- Two sources connected to two loads using three-wire system
- Sources have EQUAL magnitude and are IN PHASE

Circuit or system in which AC sources operate at the same frequency but different phases are known as polyphase.

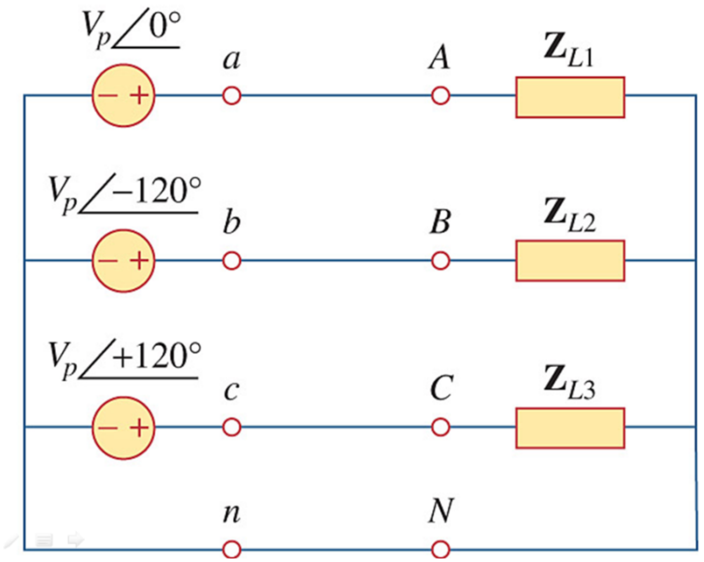


Balanced Two-phase three-wire system:

- Two sources connected to two loads using three-wire system
- Sources have EQUAL frequency but DIFFFERENT phases

Two Phase System:

- A generator consists of two coils placed perpendicular to each other
- The voltage generated by one lags the other by 90°.



Balanced Three-phase four-wire system:

- Three sources connected to 3 loads using four-wire system
- Sources have EQUAL frequency but DIFFFERENT phases

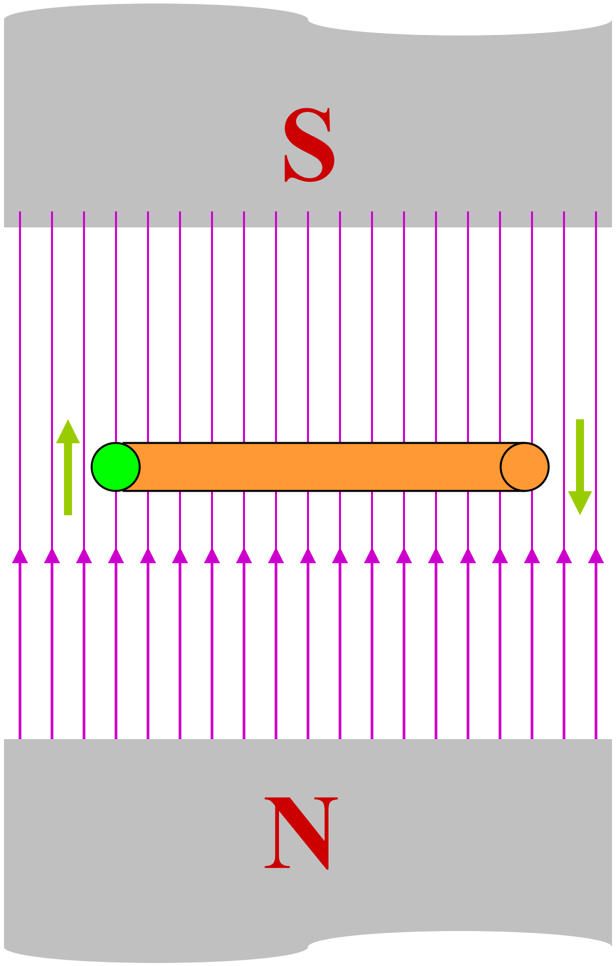
Three Phase System:

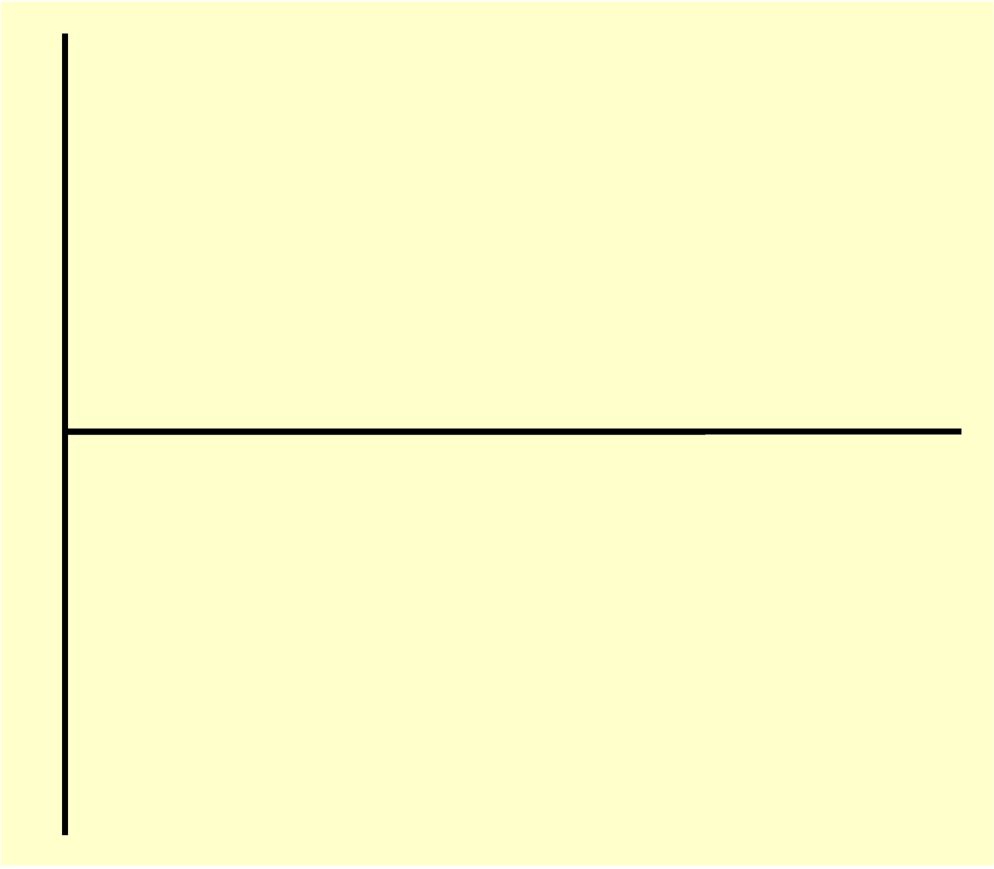
- A generator consists of <u>three coils</u> placed 120° apart.
- The voltage generated are equal in magnitude but, out of phase by 120°.
- Three phase is the most economical polyphase system.

AC Generation

- Three things must be present in order to produce electrical current:
  a) Magnetic field
  b) Conductor
  c) Relative motion
- Conductor cuts lines of magnetic flux, a voltage is induced in the conductor
- Direction and Speed are important

GENERATING A SINGLE PHASE

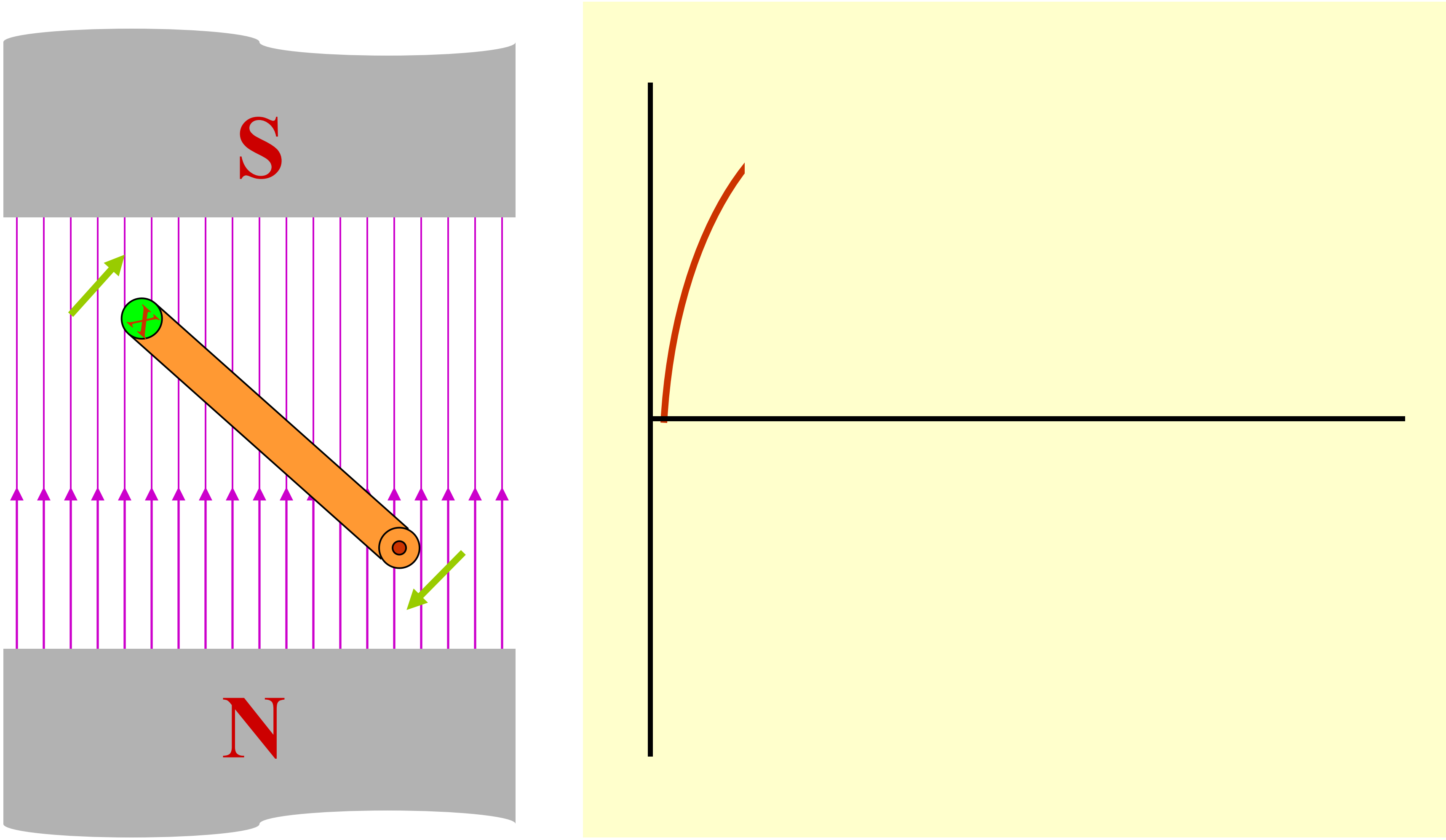




Motion is parallel to the flux.

No voltage is induced.

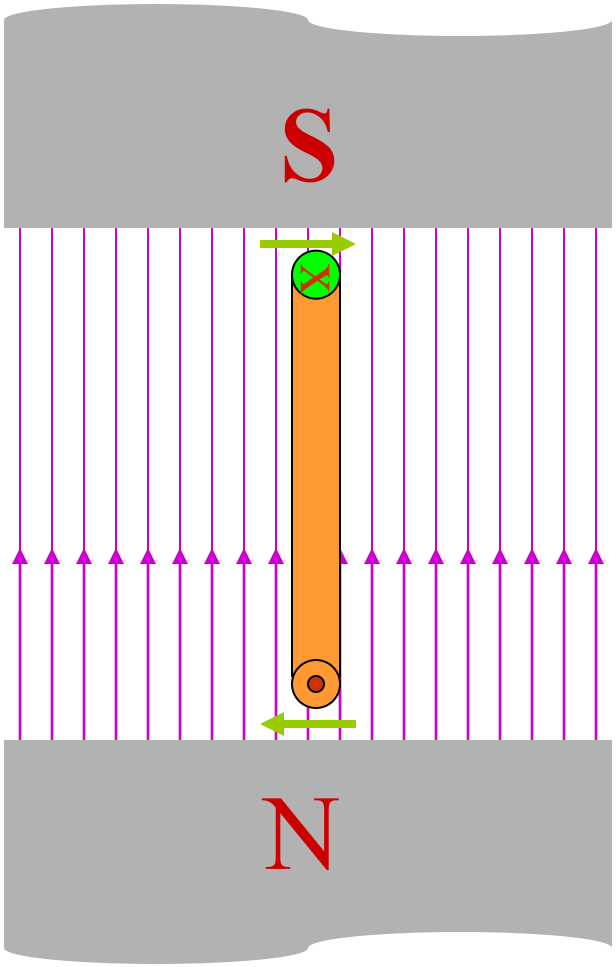
GENERATING A SINGLE PHASE

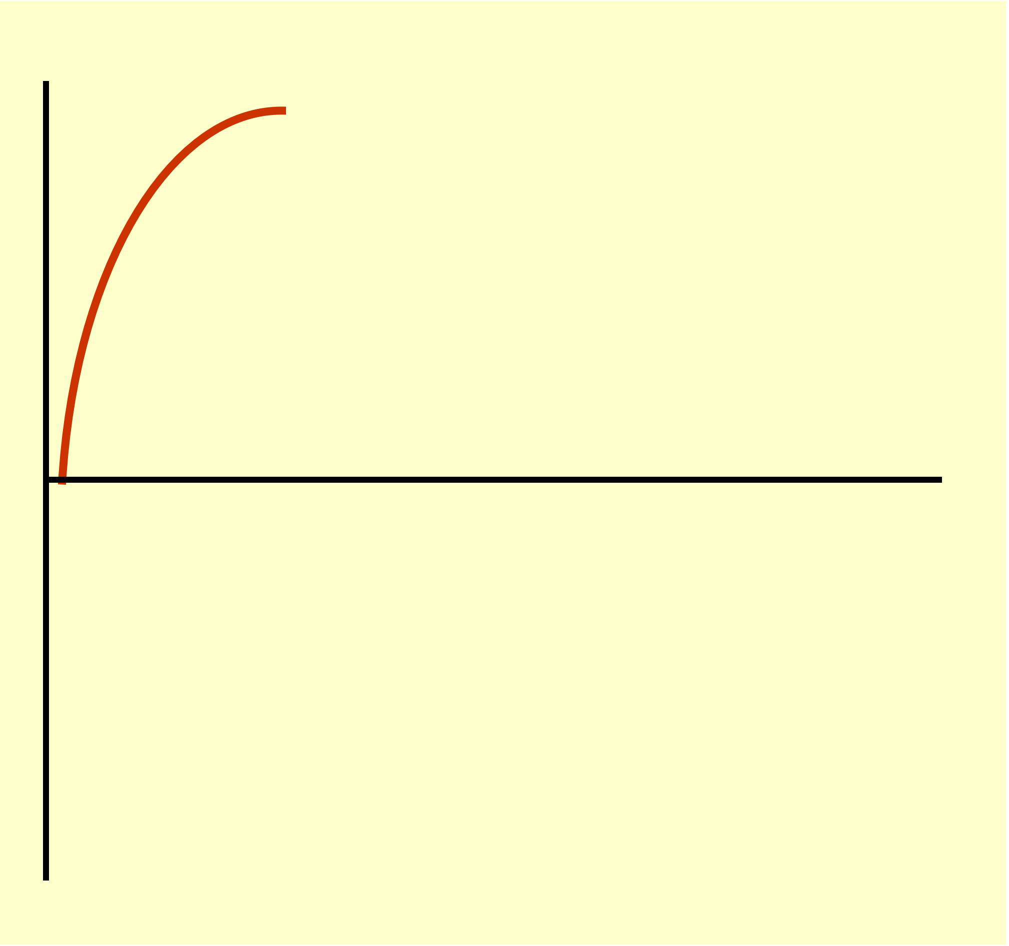


Motion is 45° to flux.

Induced voltage is 0.707 of maximum.

GENERATING A SINGLE PHASE

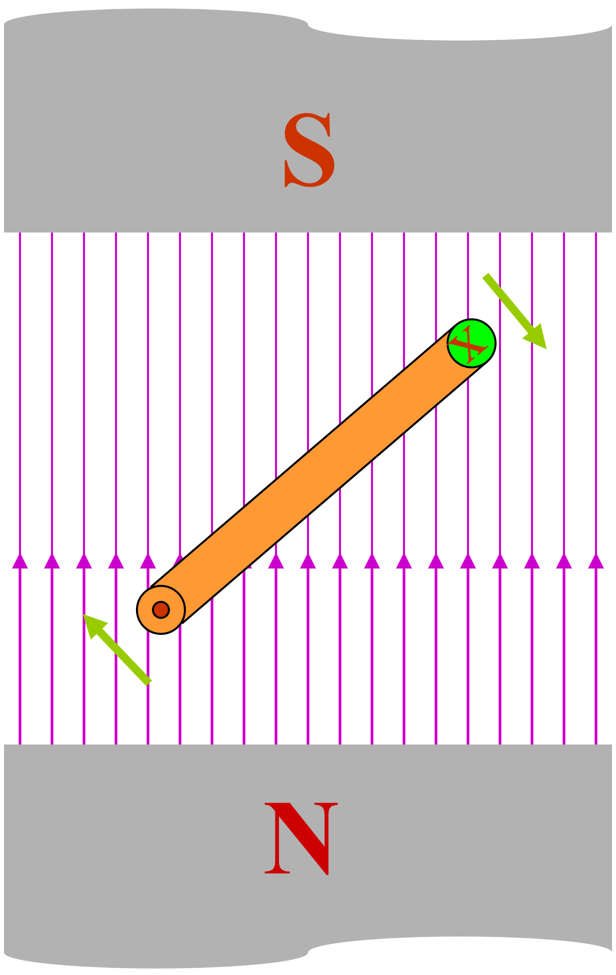


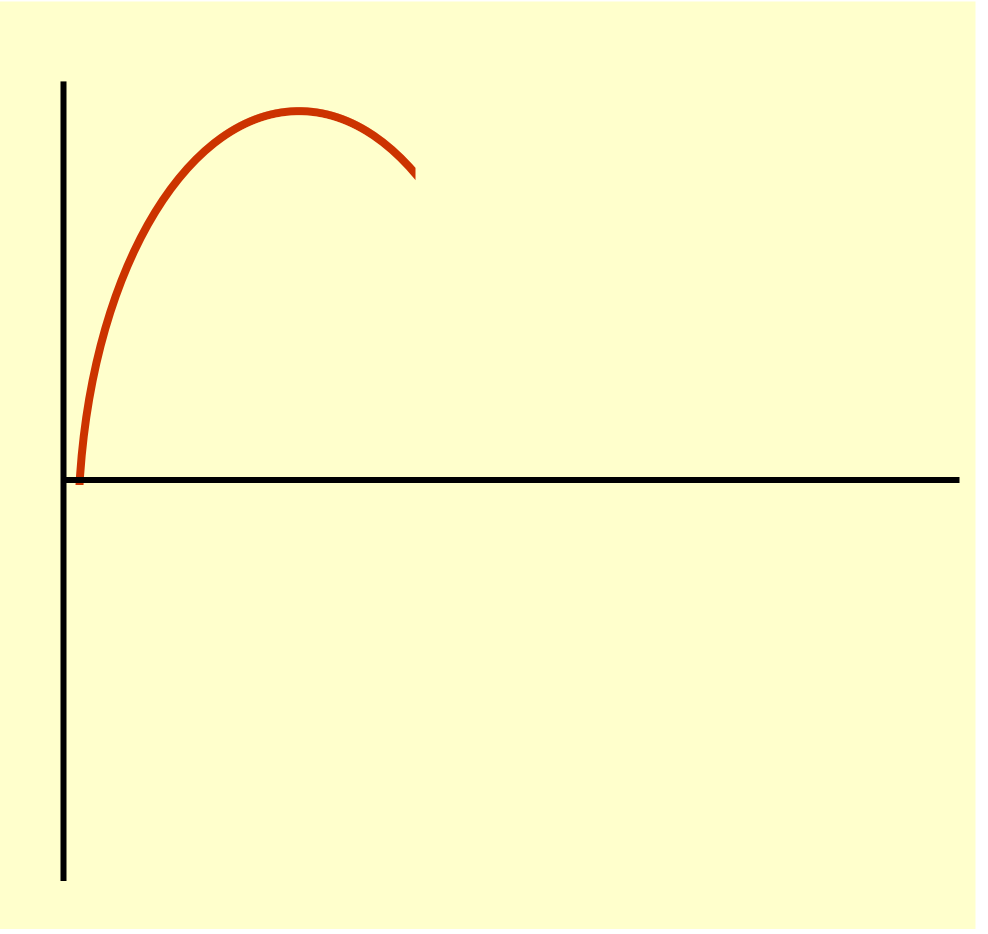


Motion is perpendicular to flux.

Induced voltage is maximum.

GENERATING A SINGLE PHASE





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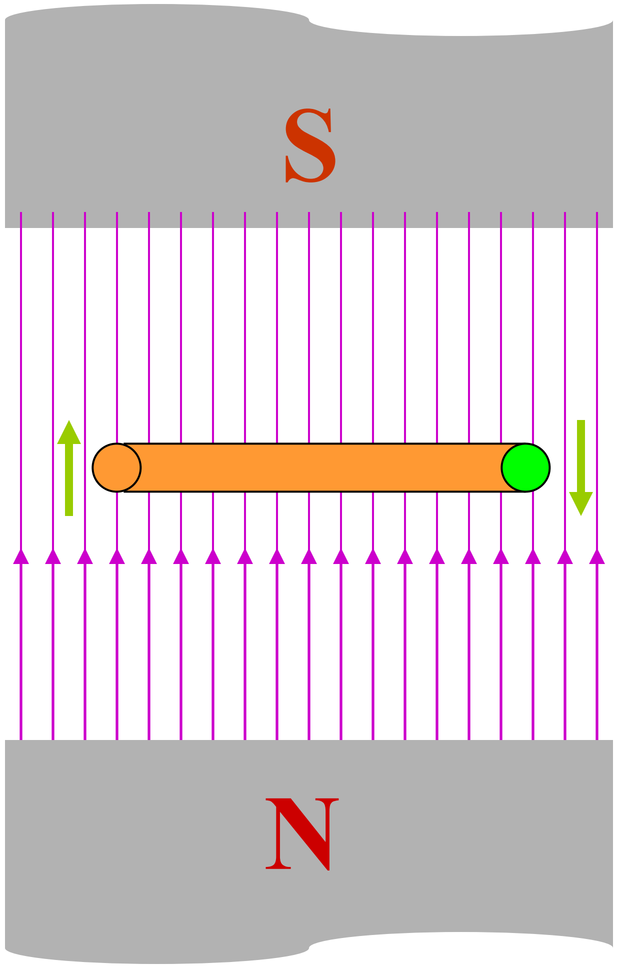
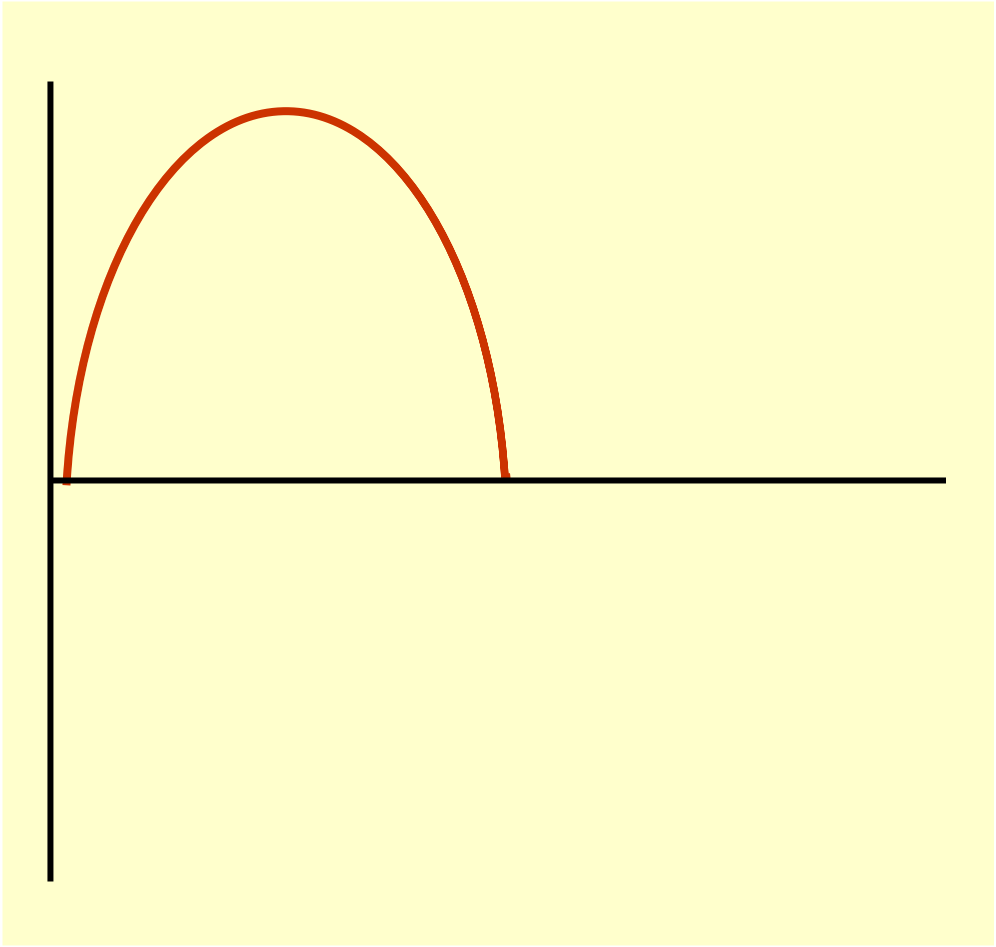
GENERATING A SINGLE PHASE

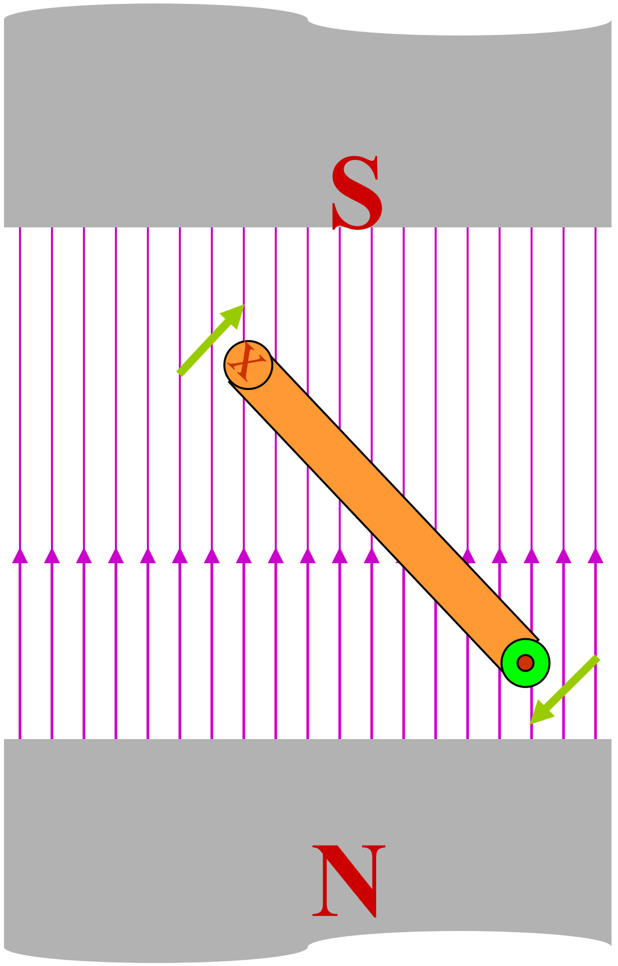


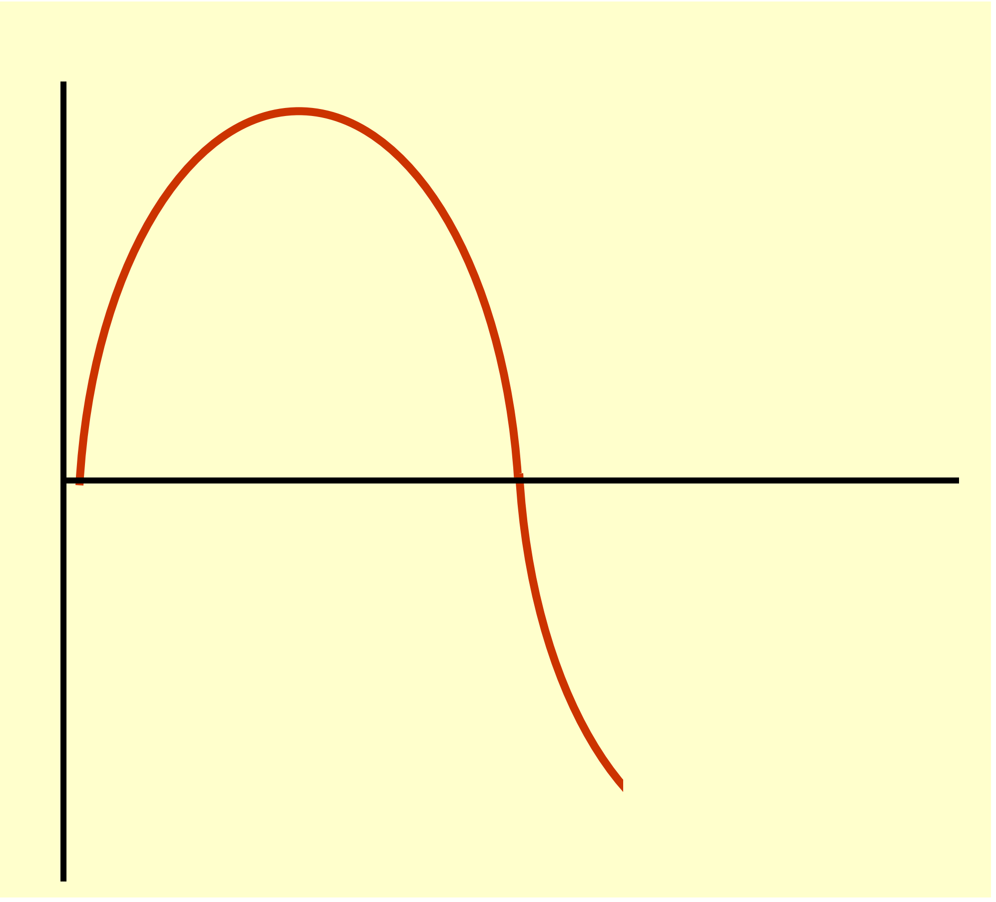


Motion is parallel to flux.

No voltage is induced.

GENERATING A SINGLE PHASE



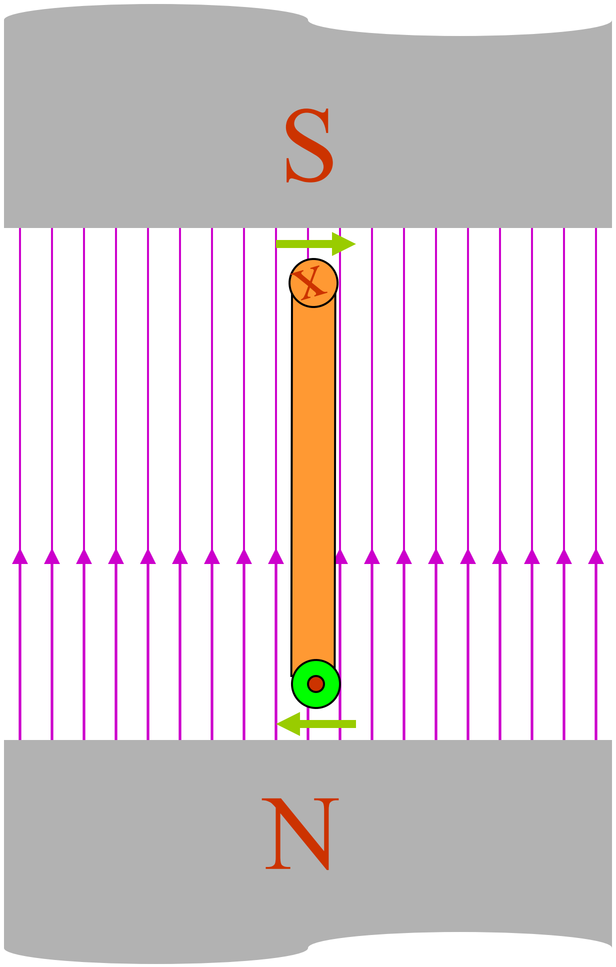


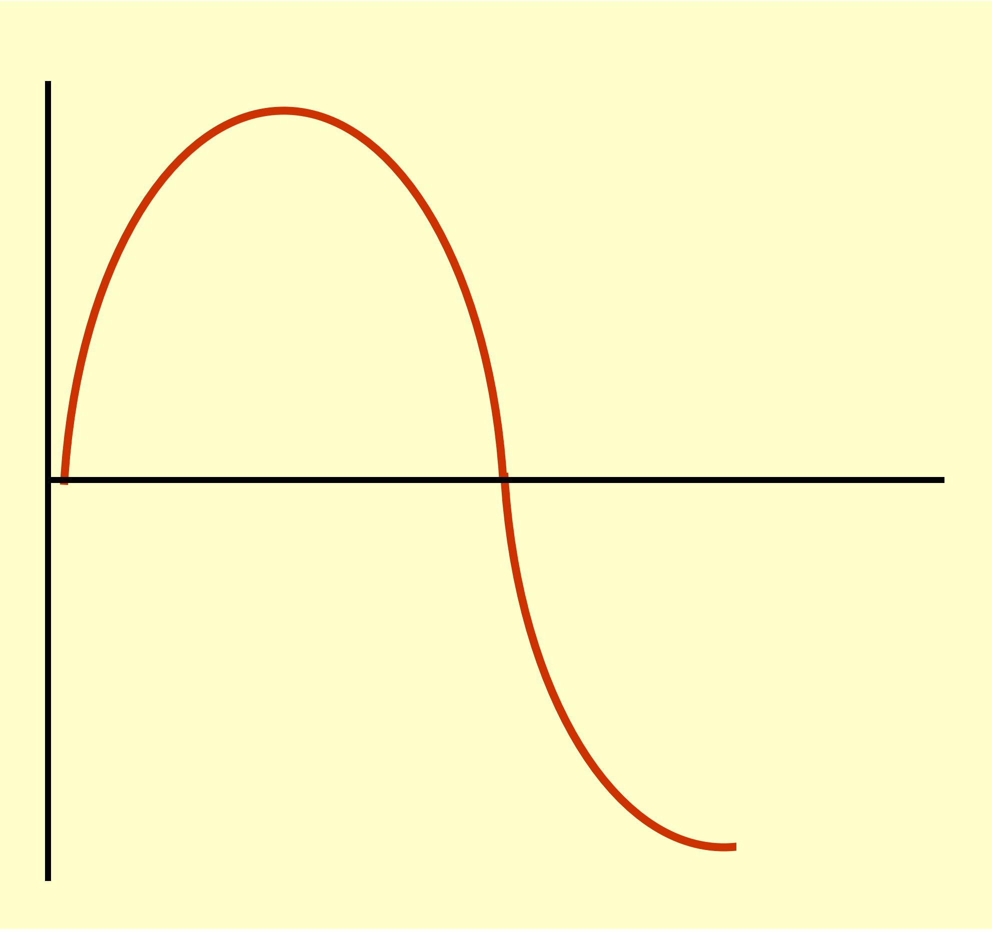
Motion is 45° to flux.

Induced voltage is 0.707 of maximum.

Notice current in the conductor has reversed.

GENERATING A SINGLE PHASE

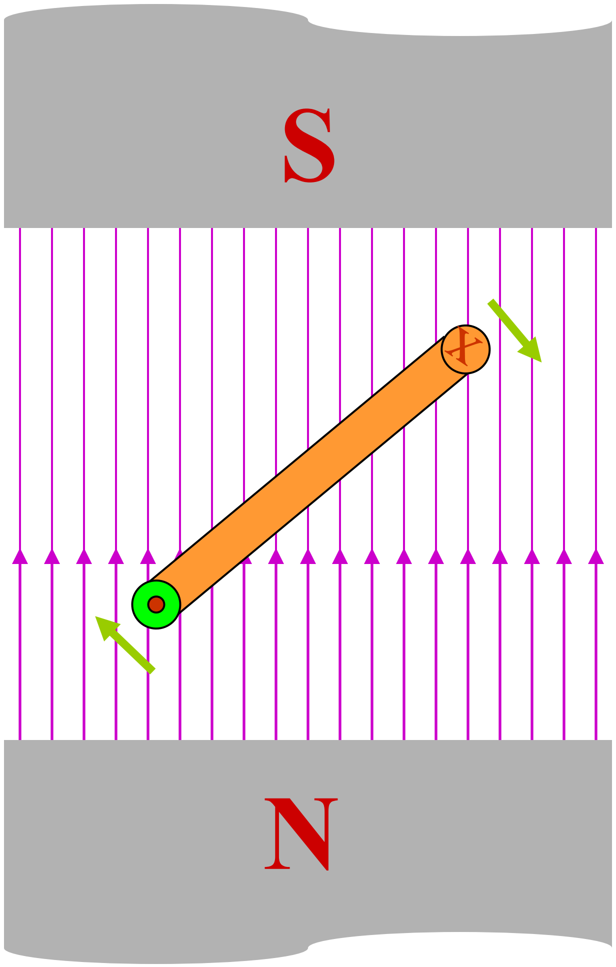


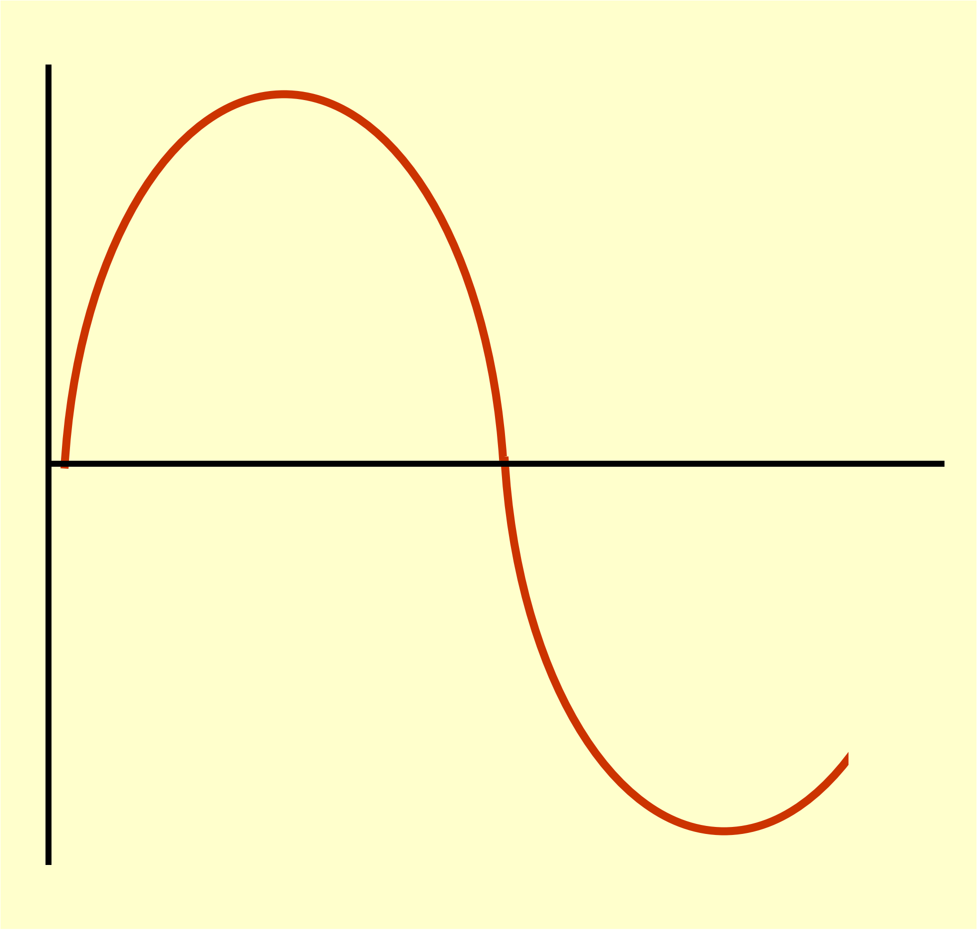


Motion is perpendicular to flux.

Induced voltage is maximum.

GENERATING A SINGLE PHASE





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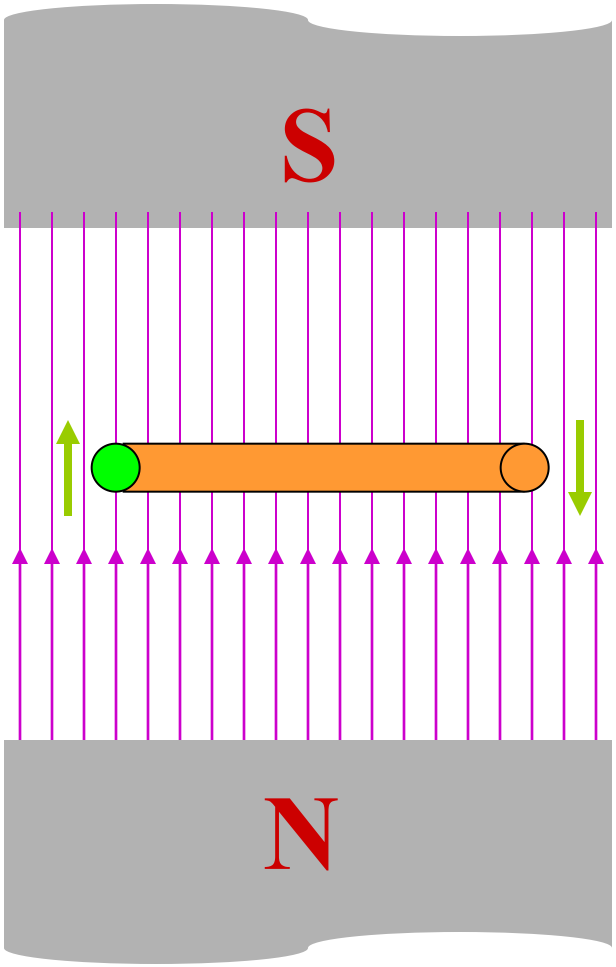
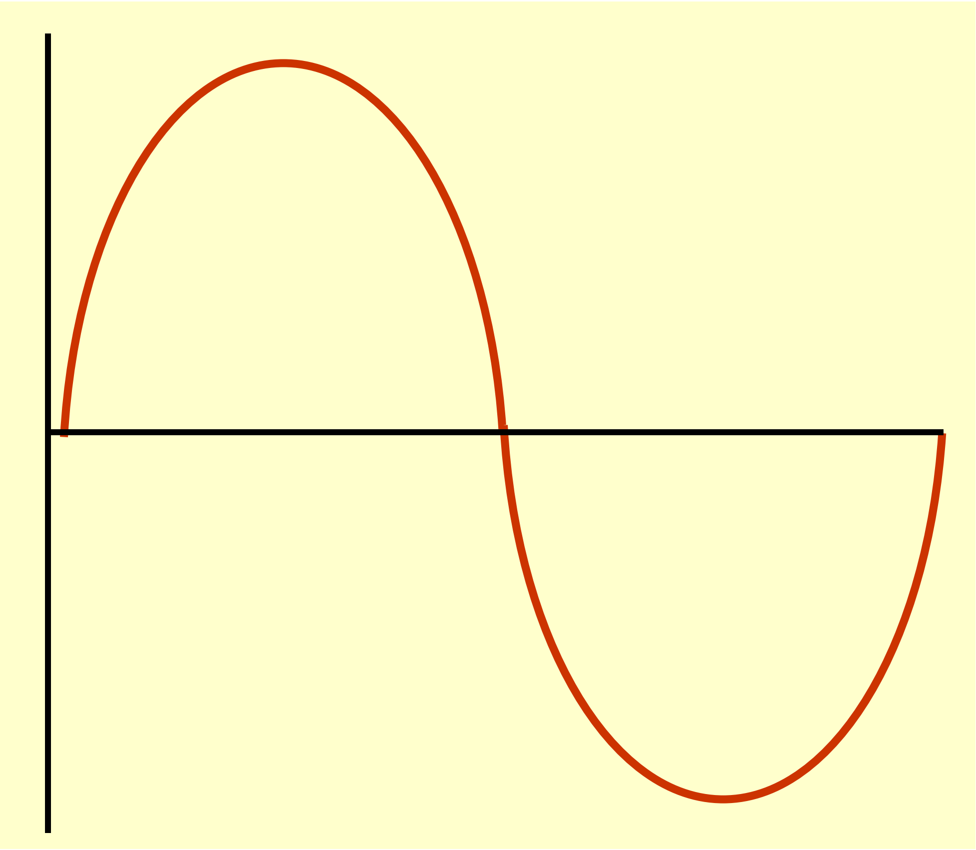
GENERATING A SINGLE PHASE





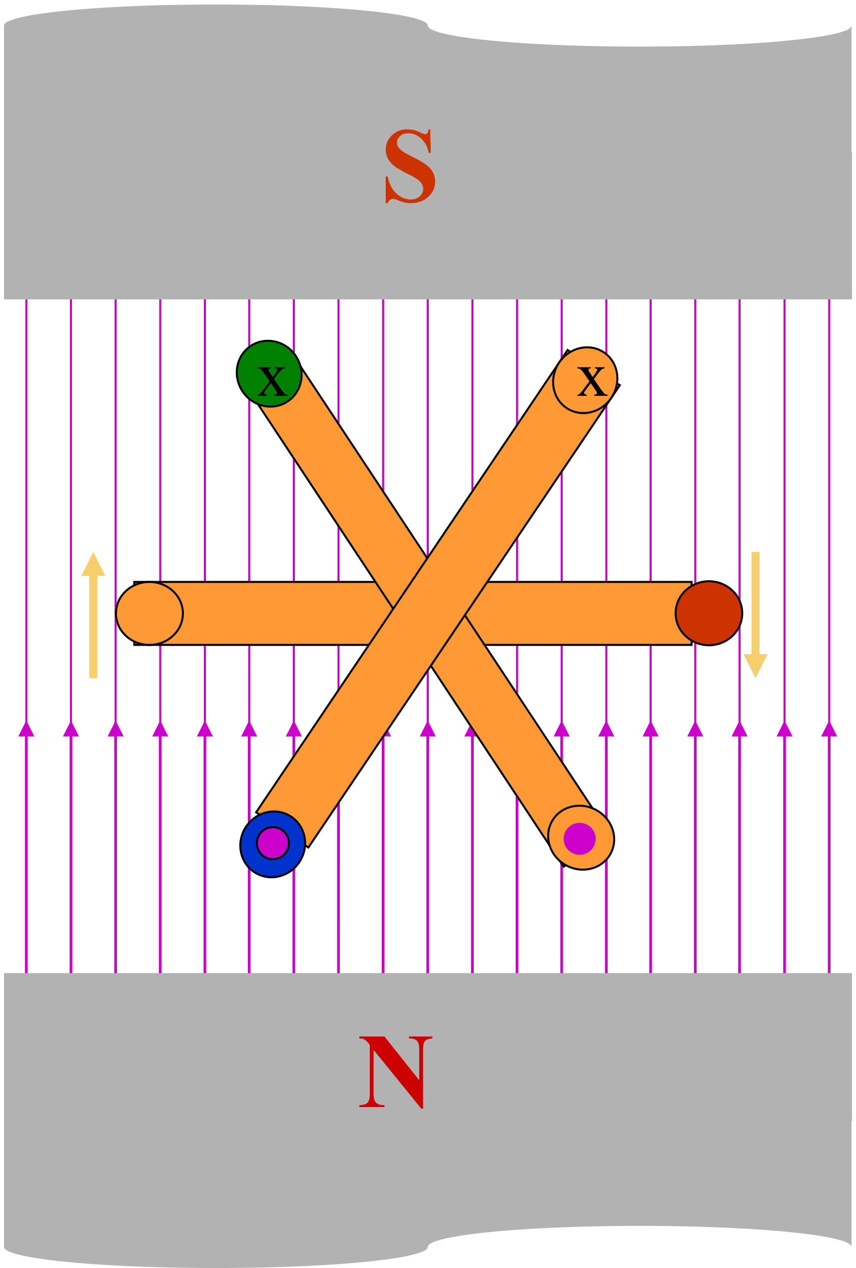
Motion is parallel to flux.

No voltage is induced.

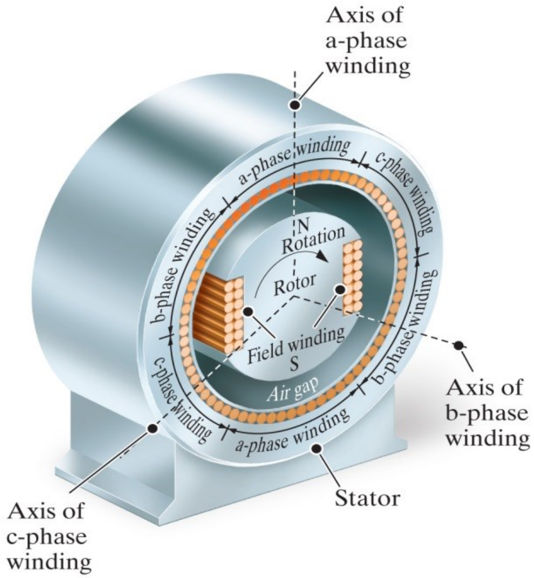
Ready to produce another cycle.

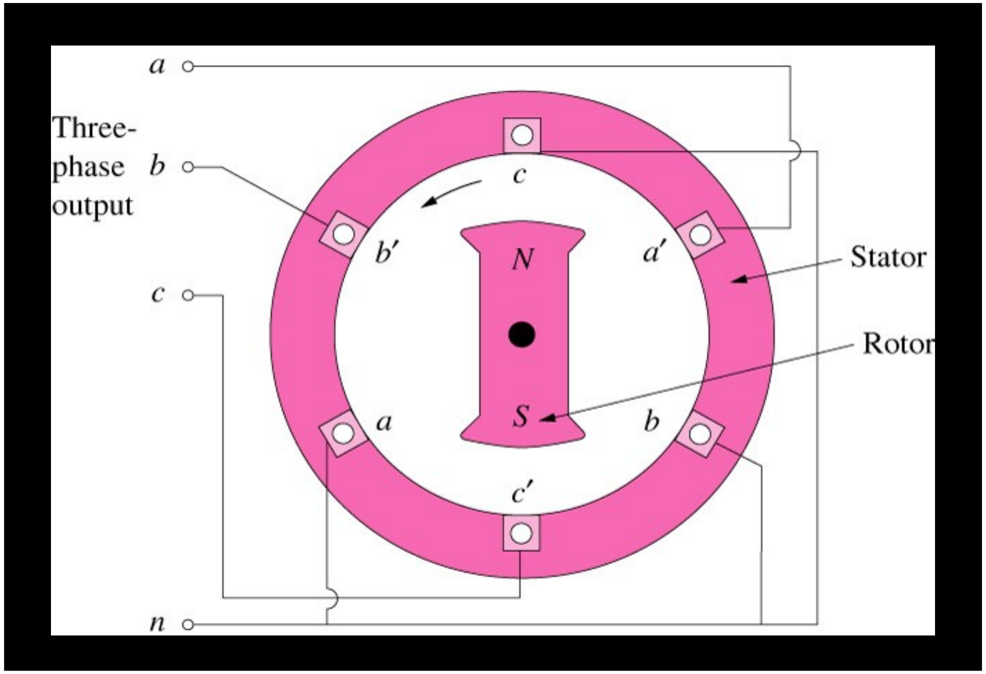
GENERATION OF THREE-PHASE AC

- Three Voltages will be induced across the coils with 120 phase difference



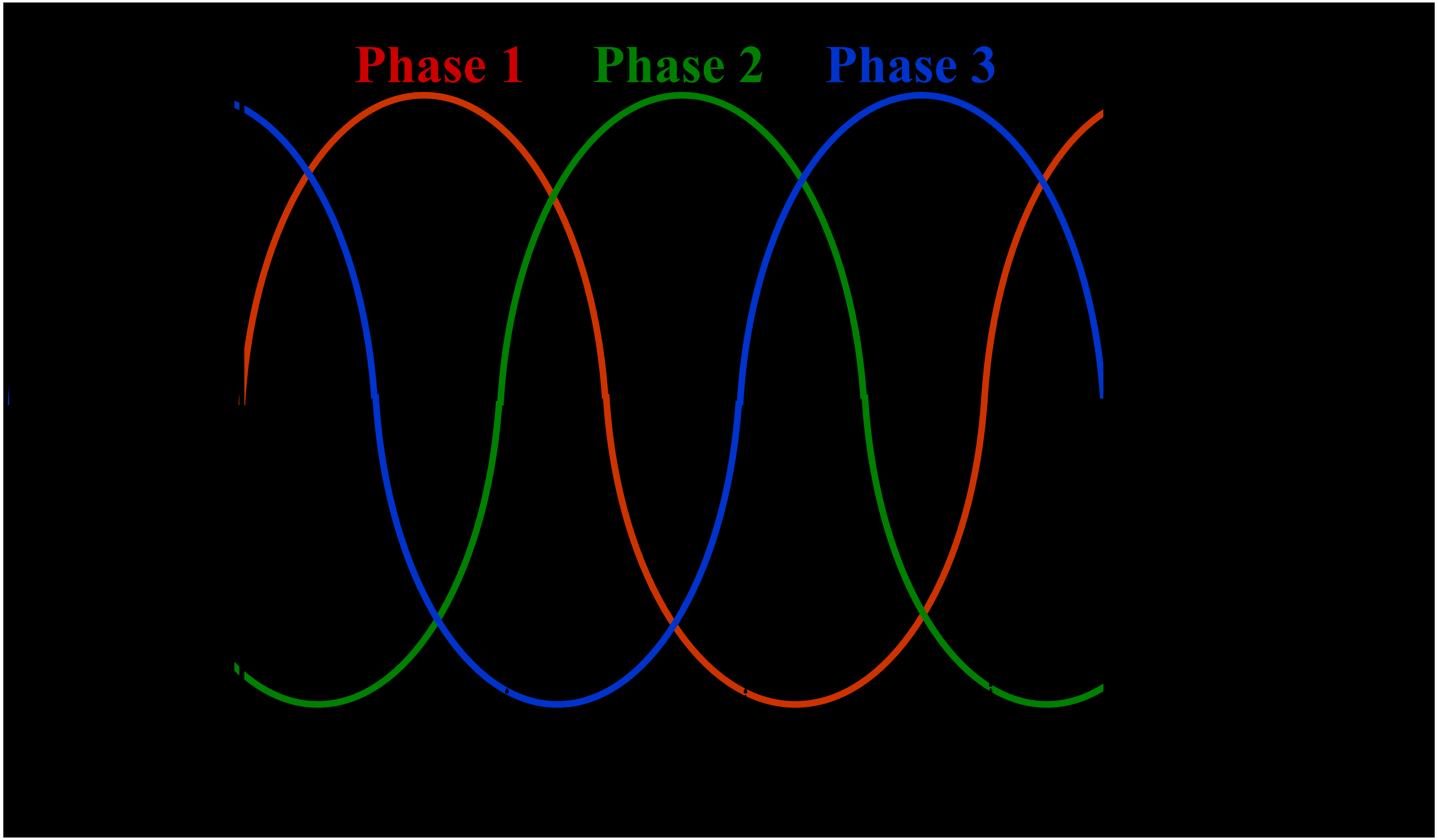
Practical THREE PHASE GENERATOR





- The generator consists of a rotating magnet (rotor) surrounded by a stationary winding (stator).
- Three separate windings or coils with terminals a-a', b-b', and c-c' are physically placed 120° apart around the stator.
- As the rotor rotates, its magnetic field cuts the flux from the three coils and induces voltages in the coils.
- The induced voltage have equal magnitude but out of phase by 120°.

THREE-PHASE WAVEFORM

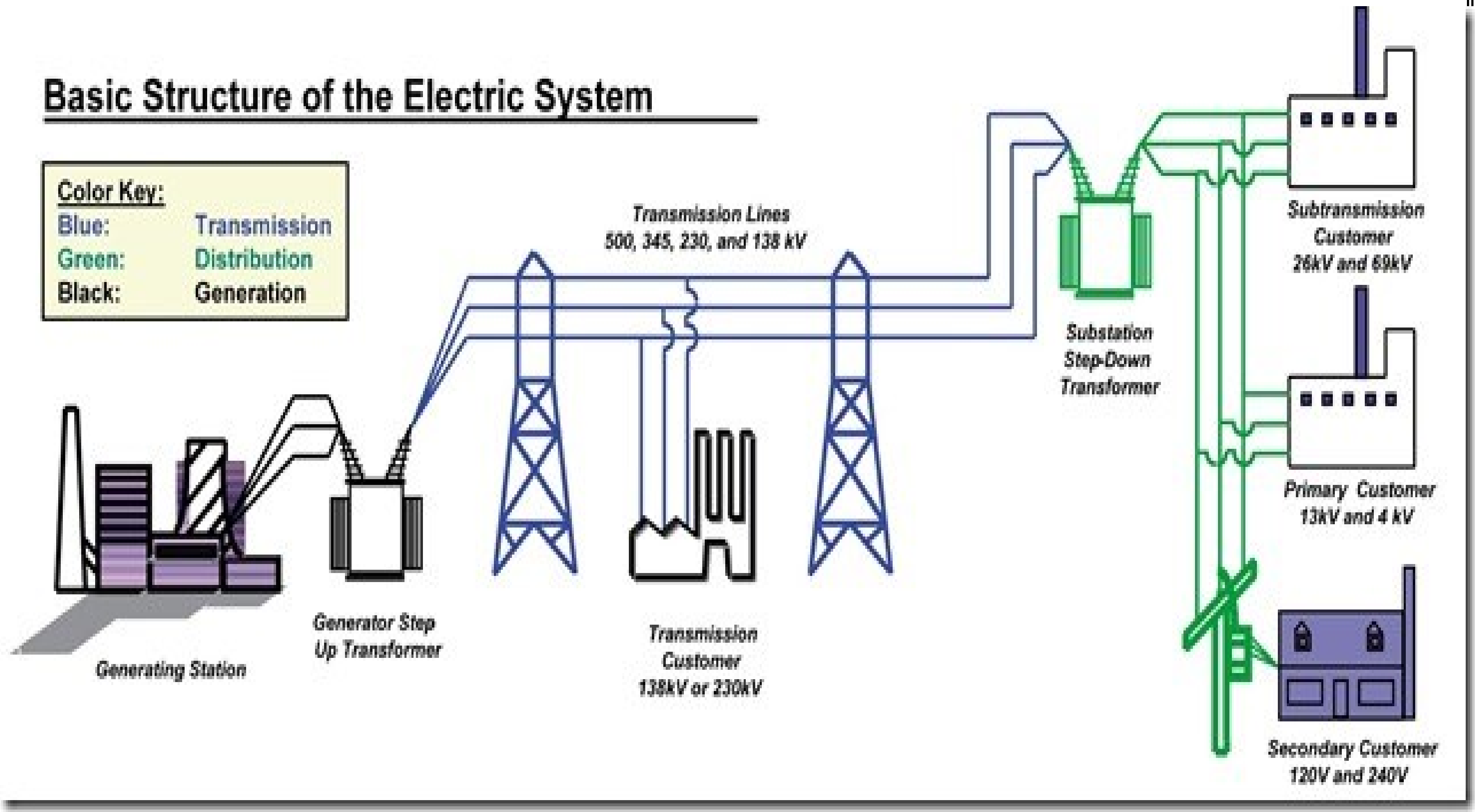


Phase 2 lags phase 1 by 120°.
Phase 2 leads phase 3 by 120°.
Phase 3 lags phase 1 by 240°.
Phase 1 leads phase 3 by 240°.

WHY WE STUDY 3 PHASE SYSTEM ?

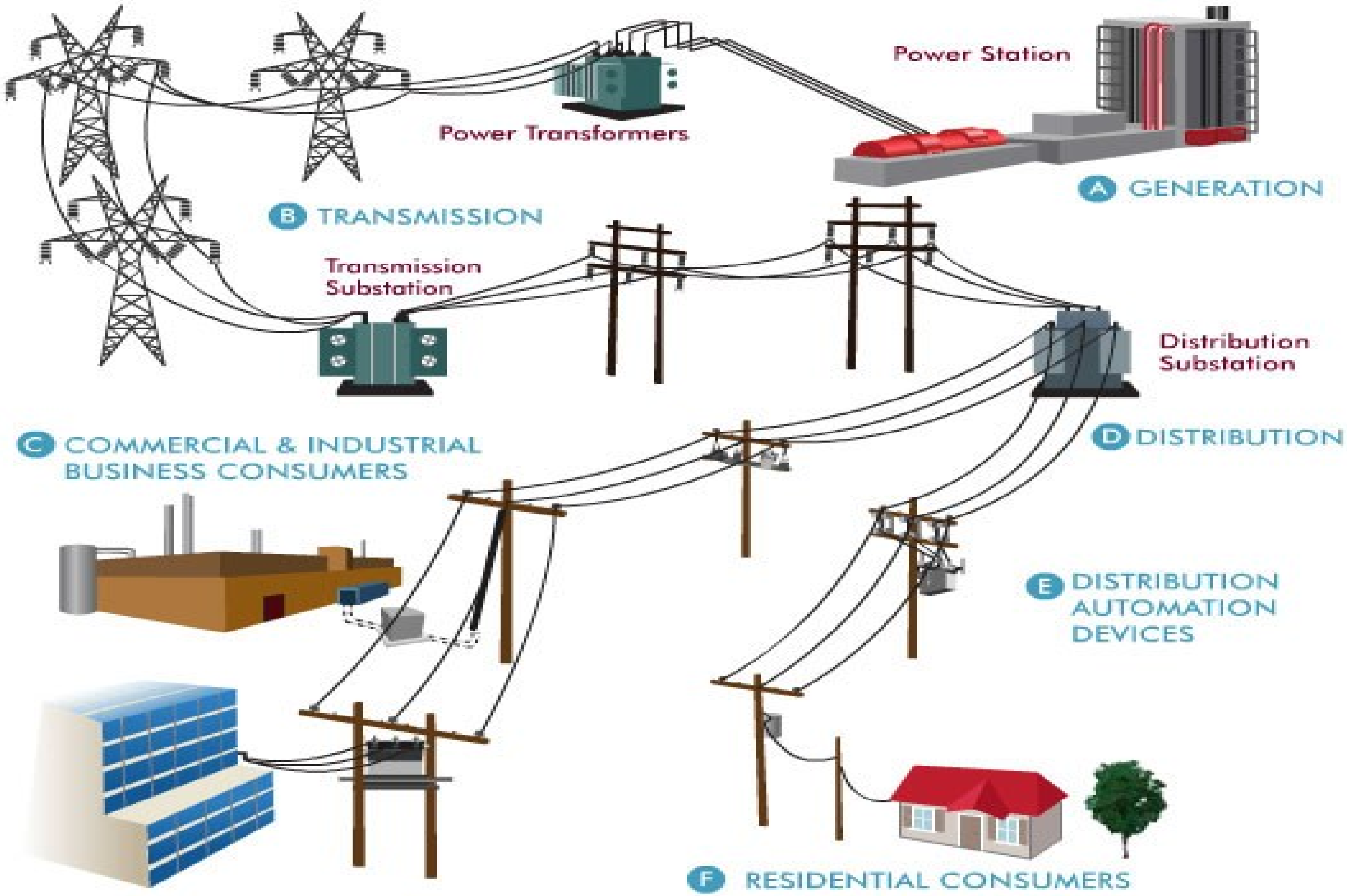
- ALL electric power system in the world used 3-phase system to GENERATE, TRANSMIT and DISTRIBUTE
  - ✓ One phase, two phase, or three phase ican be taken from three phase system rather than generated independently.
- Instantaneous power is constant (not pulsating).– thus smoother rotation of electrical machines
  - ✓ High power motors prefer a steady torque
- More economical than single phase – less wire for the same power transfer
  - ✓ The amount of wire required for a three phase system is less than required for an equivalent single phase system.

3-phase systems
Generation, Transmission and Distribution



3-phase systems

Generation, Transmission and Distribution



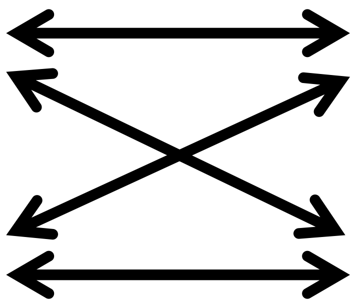
Y and Δ connections

Balanced 3-phase systems can be considered as 3 equal single phase voltage sources connected either as Y or Delta (Δ) to 3 single three loads connected as either Y or Δ

SOURCE CONNECTIONS | **LOAD CONNECTIONS**

Y connected source | **Y connected load**

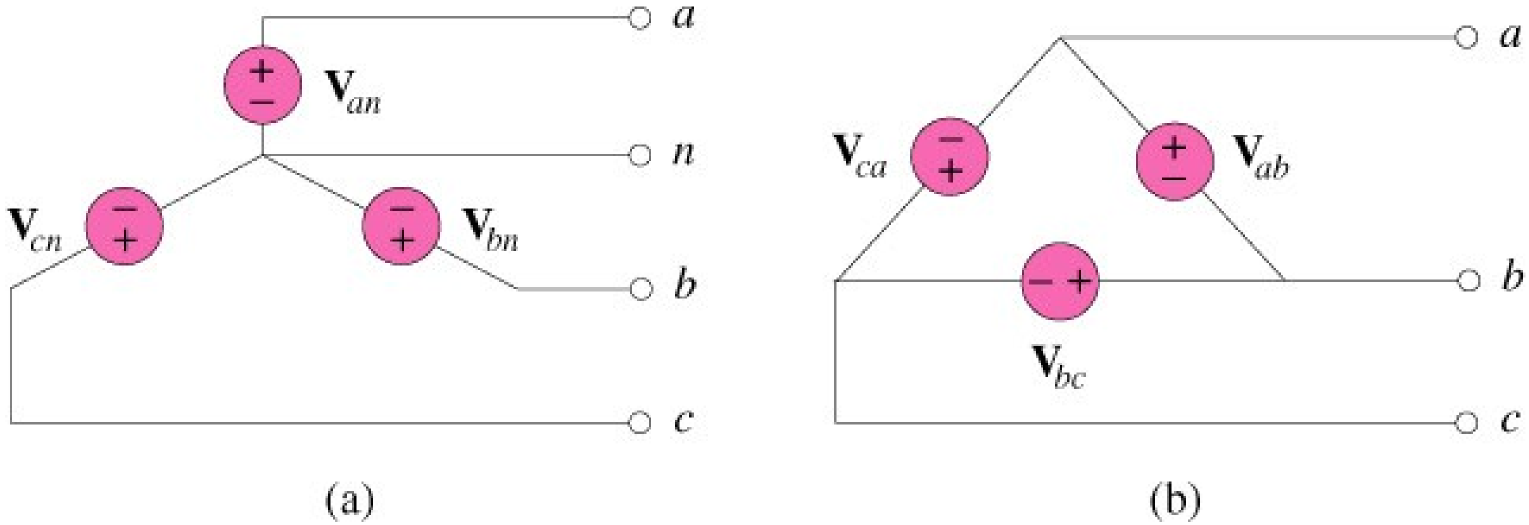
Δ connected source | **Δ connected load**



Y-Y Y-Δ Δ-Y Δ-Δ

Balance Three-Phase Voltages

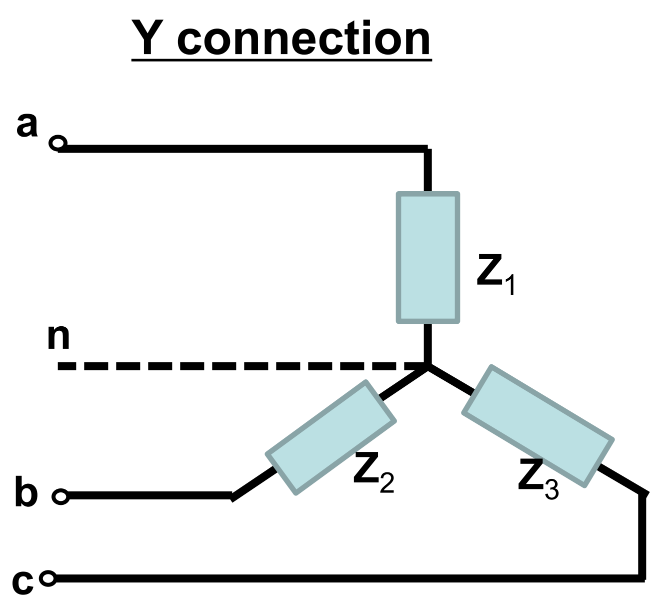
Two possible configurations:

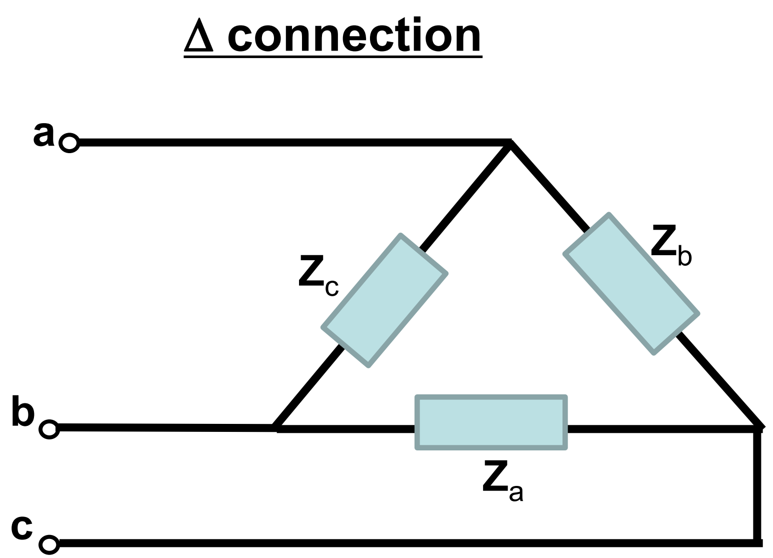


Three-phase voltage sources: (a) Y-connected ; (b) Δ-connected

Balanced 3-phase systems

LOAD CONNECTIONS





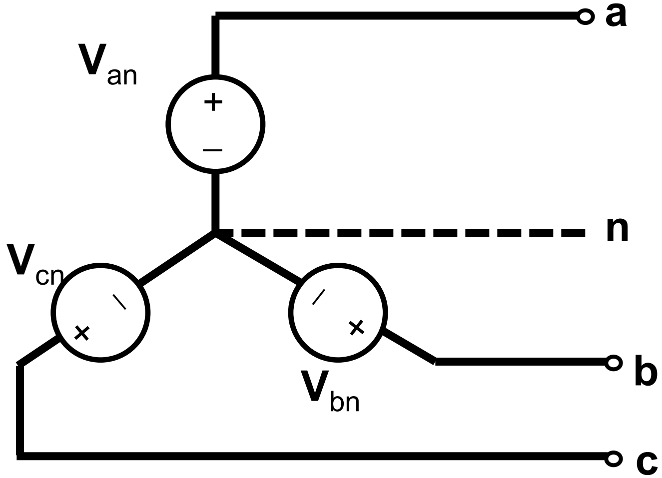
Balanced load:

$Z_1 = Z_2 = Z_3 = Z_Y$ $\quad$ $Z_a = Z_b = Z_c = Z_\Delta$ $\quad$ $Z_Y = \frac{Z_\Delta}{3}$

Unbalanced load: each phase load may not be the same.

Phase Sequence

The ***phase sequence*** is the time order in which the voltages pass through their respective maximum values.

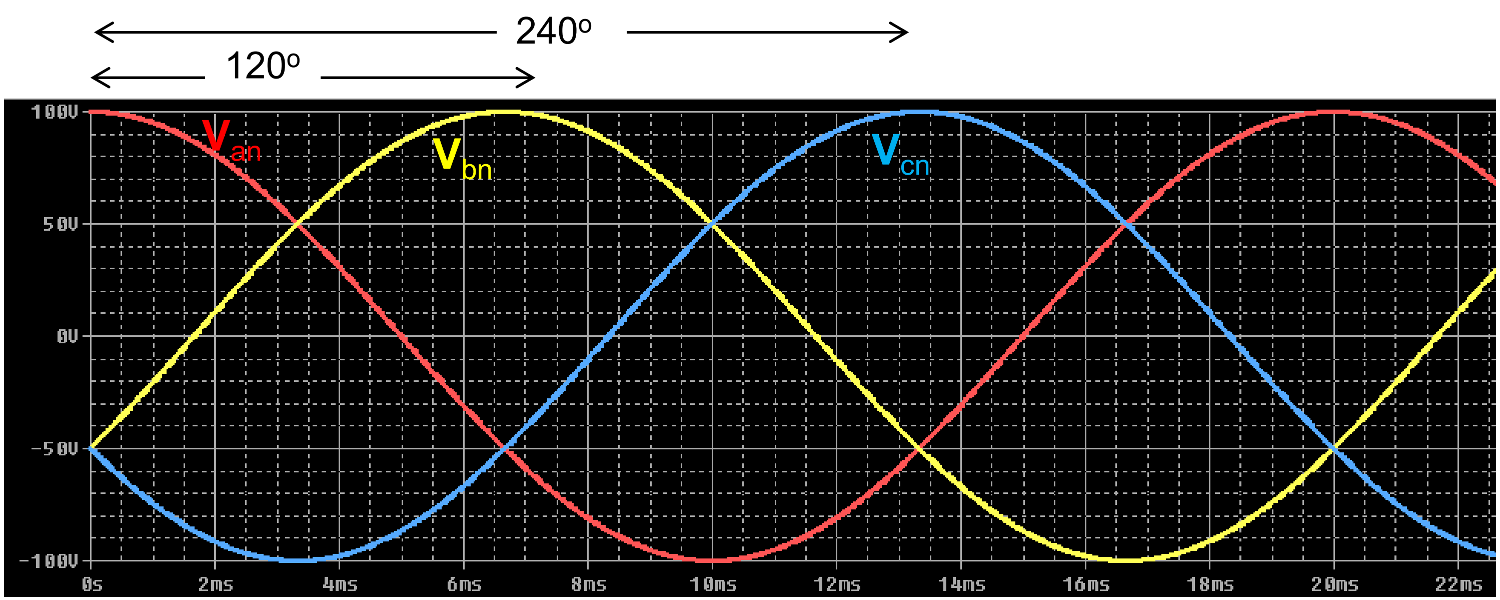


$$v_{an}(t) = \sqrt{2}V_p \cos(\omega t) \Rightarrow \mathbf{V}_{an} = V_p\angle 0^o$$

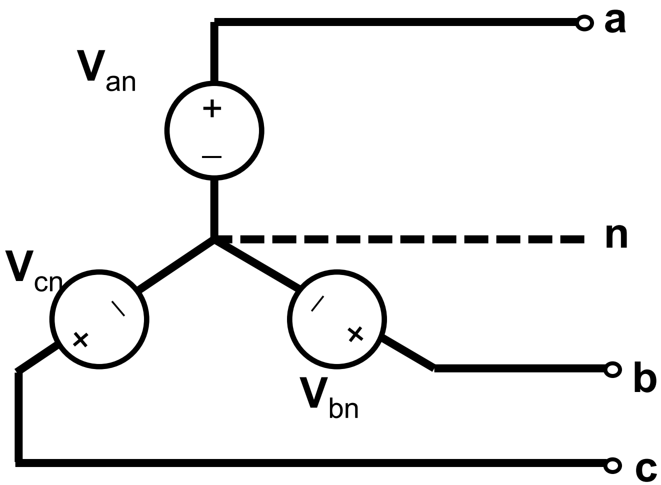
$$v_{bn}(t) = \sqrt{2}V_p \cos(\omega t - 120^o) \Rightarrow \mathbf{V}_{bn} = V_p\angle -120^o$$

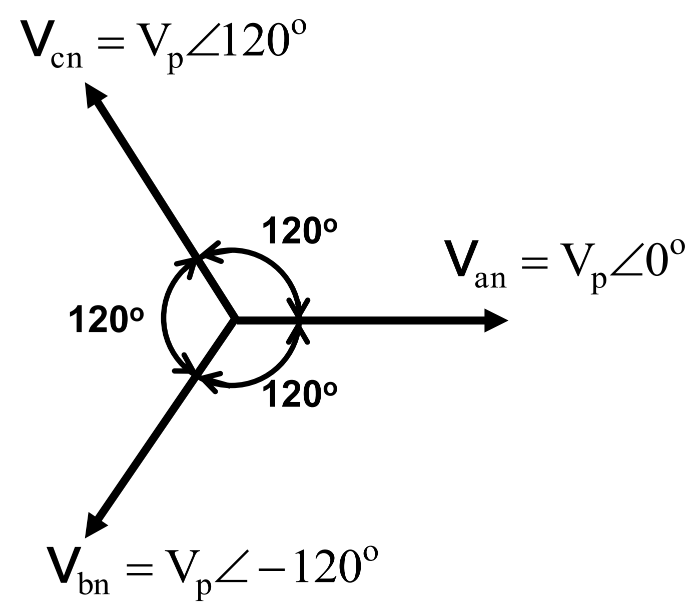
$$v_{cn}(t) = \sqrt{2}V_p \cos(\omega t + 120^o) \Rightarrow \mathbf{V}_{cn} = V_p\angle 120^o$$

RMS phasors !



Phase Sequence





Phase sequence : **V_{an}** leads **V_{bn}** by 120^o and **V_{bn}** leads **V_{cn}** by 120^o

→This is a known as **abc sequence** or **positive sequence**

Phase Sequence

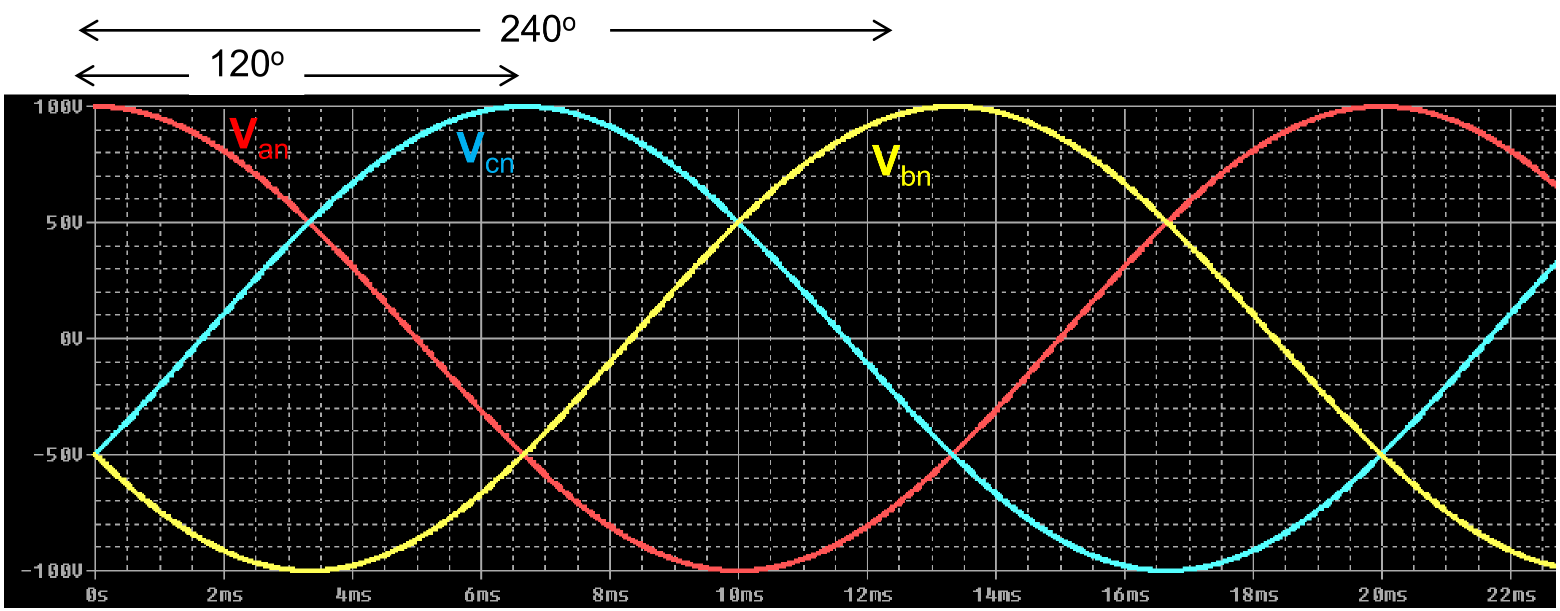
What if different phase sequence?

$$v_{an}(t) = \sqrt{2}V_p \cos(\omega t) \Rightarrow \mathbf{V}_{an} = V_p \angle 0^o$$

$$v_{cn}(t) = \sqrt{2}V_p \cos(\omega t - 120^o) \Rightarrow \mathbf{V}_{cn} = V_p \angle -120^o$$

$$v_{bn}(t) = \sqrt{2}V_p \cos(\omega t + 120^o) \Rightarrow \mathbf{V}_{bn} = V_p \angle 120^o$$

RMS phasors !



Phase Sequence

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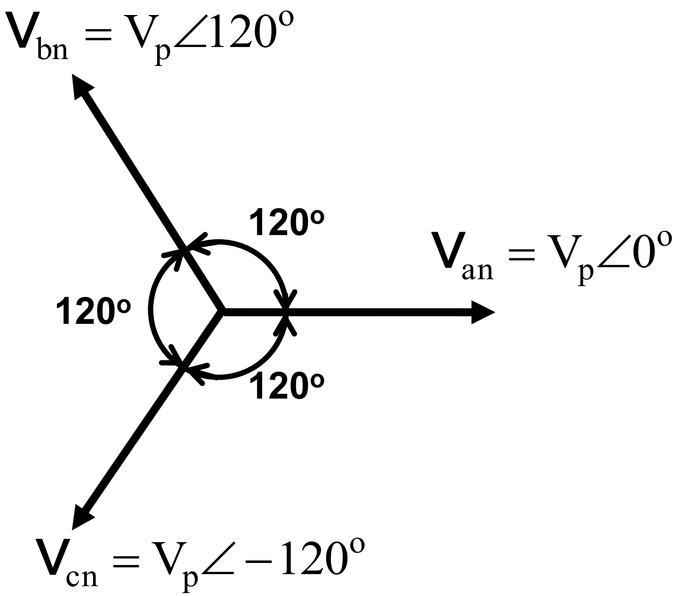


Phase sequence : $\mathbf{V}_{an}$ leads $\mathbf{V}_{cn}$ by 120^o and $\mathbf{V}_{cn}$ leads $\mathbf{V}_{bn}$ by 120^o

→This is a known as **acb sequence** or **negative sequence**

Example 1

Determine the phase sequence of the set of voltages.

$$v_{an} = 200\cos(\omega t + 10^\circ)$$

$$v_{bn} = 200\cos(\omega t - 230^\circ)$$

$$v_{cn} = 200\cos(\omega t - 110^\circ)$$

Solution:

The voltages can be expressed in phasor form as

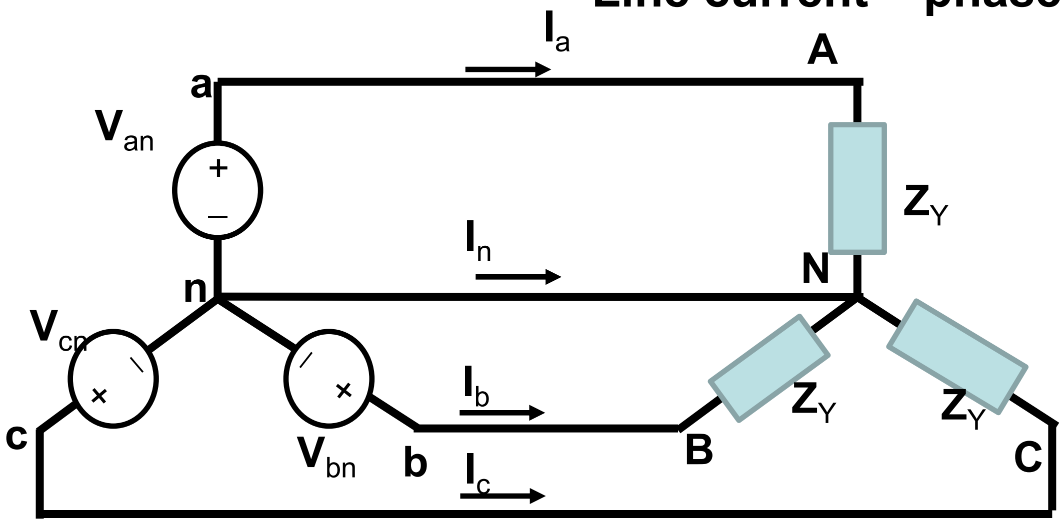
$$V_{an} = 200\angle 10^\circ \text{ V}$$
$$V_{bn} = 200\angle -230^\circ \text{ V}$$
$$V_{cn} = 200\angle -110^\circ \text{ V}$$

We notice that $\mathbf{V_{an}}$ leads $\mathbf{V_{cn}}$ by 120° and $\mathbf{V_{cn}}$ in turn leads $\mathbf{V_{bn}}$ by 120°.

Hence, we have an **acb** sequence.

Balanced 3-phase Y-Y

Line current = phase current



$$V_{an} = V_p\angle 0^o$$

$$V_{bn} = V_p\angle -120^o$$

$$V_{cn} = V_p\angle 120^o$$

Phase voltages

measured between the neutral and any line (line to neutral voltage)

$$I_a = \frac{V_p\angle 0^o}{Z_Y}$$

$$I_b = \frac{V_p\angle -120^o}{Z_Y}$$

$$I_c = \frac{V_p\angle 120^o}{Z_Y}$$

line currents

$$\therefore I_a + I_b + I_c = I_n = 0$$

The wire connecting n and N can be removed !

$$\mathbf{V}_{ab} = V_a - V_b = V_a - V_b + V_n - V_n =$$

$$= V_{an} + V_{nb} = V_p\angle 0^o + V_p\angle 60^o$$

$$= \sqrt{3}V_p\angle 30^o$$

$$V_{bc} = V_{bn} + V_{nc}$$

$$= \sqrt{3}V_p\angle -90^o$$

$$V_{ca} = V_{cn} + V_{na}$$

$$= \sqrt{3}V_p\angle 150^o$$

line-line voltages OR Line voltages

Balanced 3-phase systems

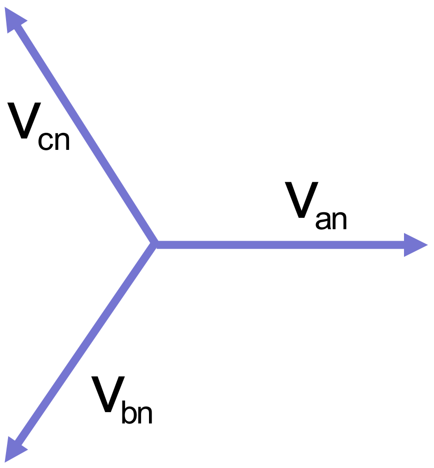
Balanced Y-Y Connection

$$\mathbf{V}_{ab} = V_{an} + V_{nb}$$
$$= V_p\angle 0^o + V_p\angle 60^o$$
$$= \sqrt{3}V_p\angle 30^o$$

Balanced 3-phase systems

Balanced Y-Y Connection

$$
\begin{aligned}
V_{ab} &= V_{an} + V_{nb} \\
&= V_p\angle 0^o + V_p\angle 60^o \\
&= \sqrt{3}V_p\angle 30^o
\end{aligned}
$$



Balanced 3-phase systems

Balanced Y-Y Connection

$$
\begin{aligned}
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&= V_p\angle 0^o + V_p\angle 60^o \\
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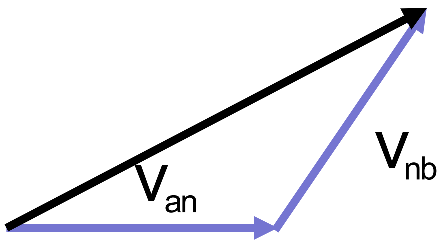

Balanced 3-phase systems

Balanced Y-Y Connection

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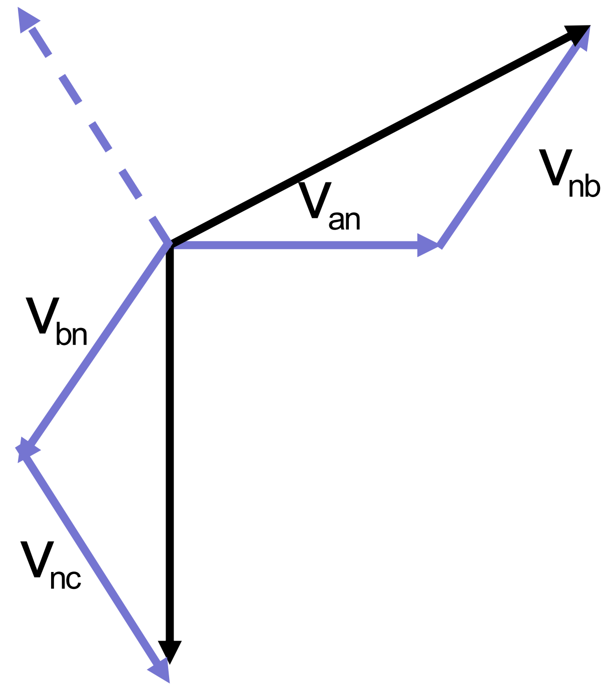
$$V_{bc} = V_{bn} + V_{nc}$$
$$= \sqrt{3}V_p\angle -90^o$$



Balanced 3-phase systems

Balanced Y-Y Connection

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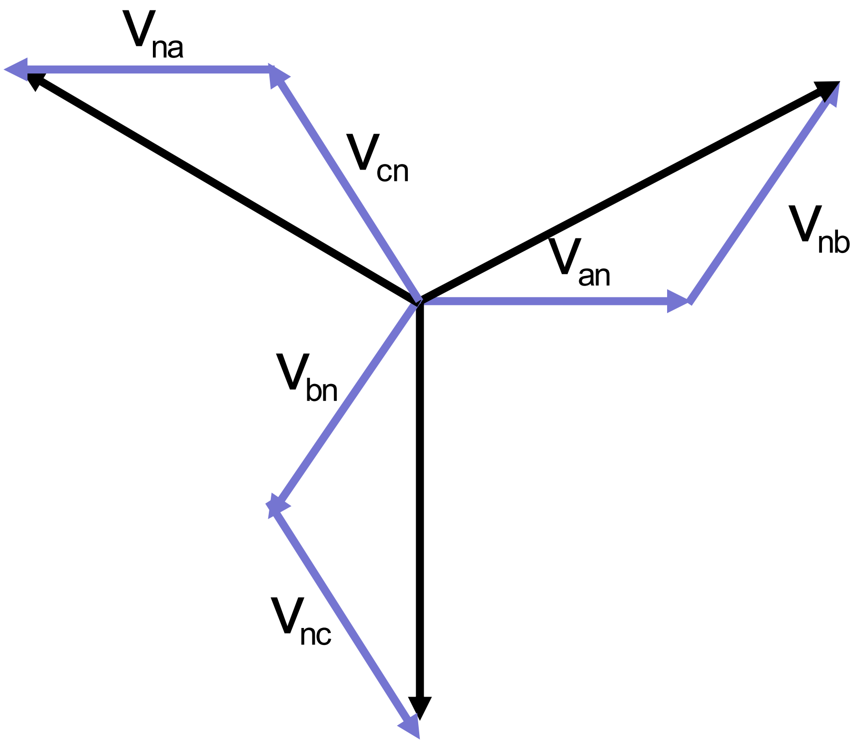

Balanced 3-phase systems

Balanced Y-Y Connection

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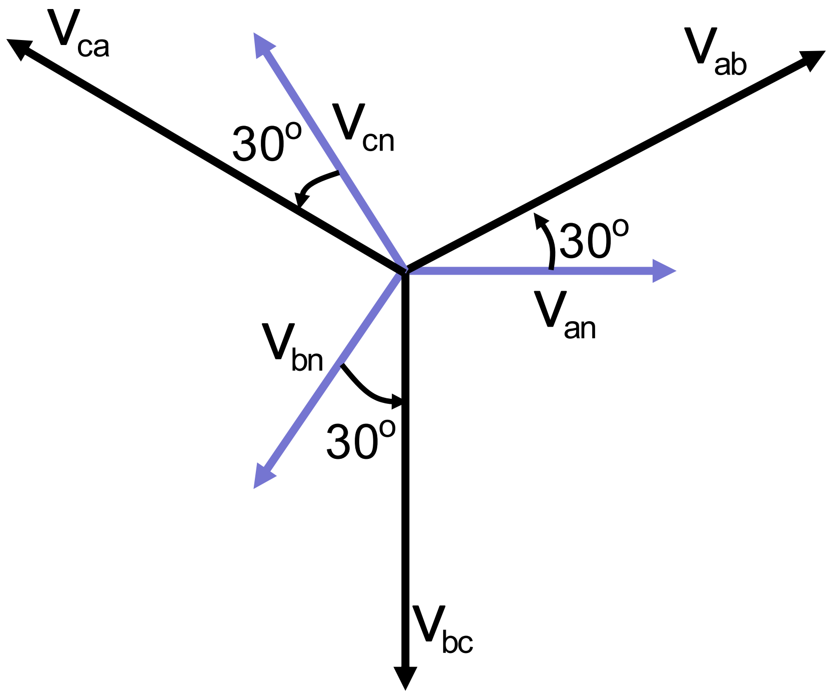
$$\mathbf{V}_{ca} = V_{cn} + V_{na}$$
$$= \sqrt{3}V_p\angle 150^o$$



$$V_L = \sqrt{3}V_p$$

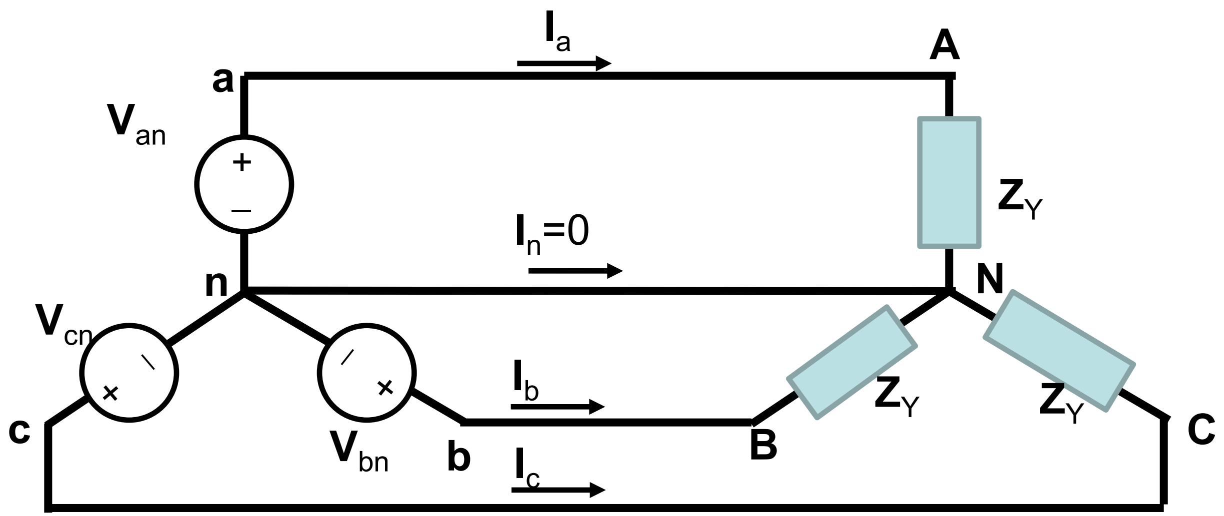
where $V_L = |\mathbf{V}_{ab}| = |\mathbf{V}_{bc}| = |\mathbf{V}_{ca}|$ and $V_p = |\mathbf{V}_{an}| = |\mathbf{V}_{bn}| = |\mathbf{V}_{cn}|$

Line voltage LEADS phase voltage by 30^o

Balanced 3-phase systems

Balanced Y-Y Connection

For a **balanced Y-Y** connection, analysis can be performed using an equivalent per-phase circuit: e.g. for phase A:



Balanced 3-phase systems

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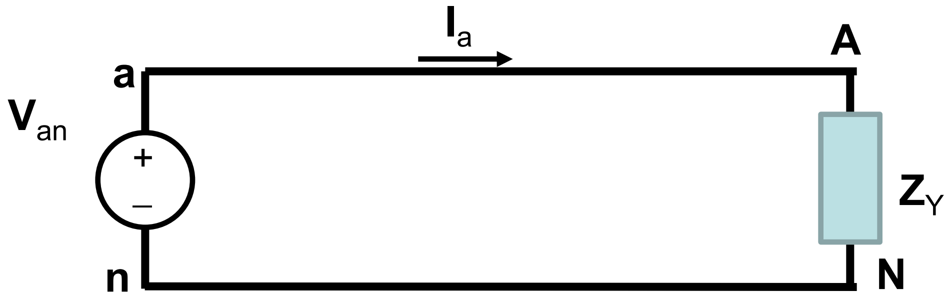


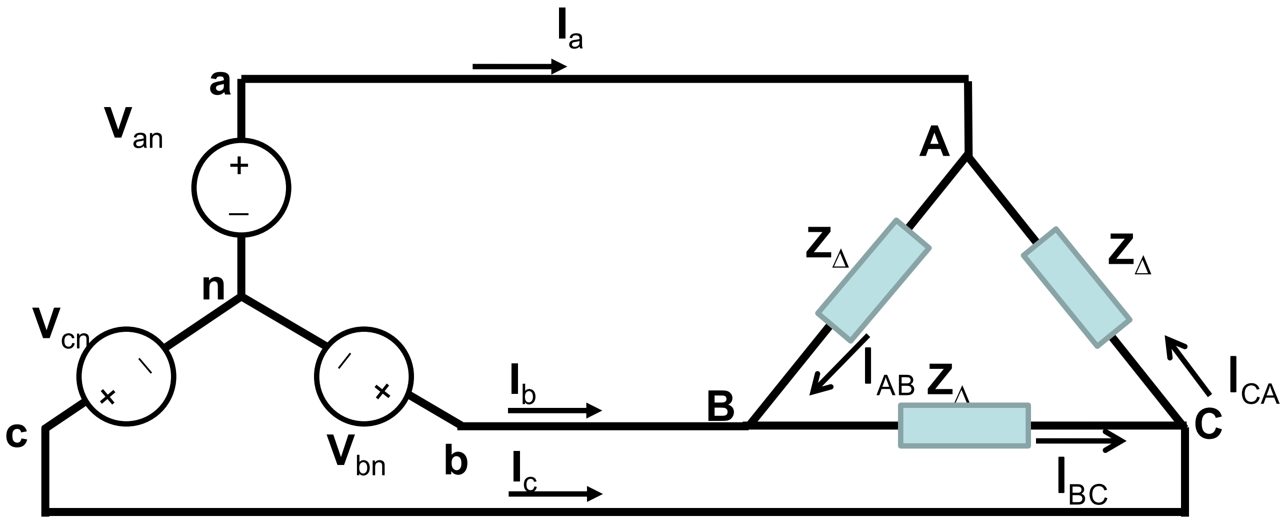
$$I_a = \frac{V_{an}}{Z_Y}$$

Based on the sequence, the other line currents can be obtained from:

$$I_b = I_a \angle -120^o \qquad I_c = I_a \angle 120^o$$

Balanced 3-phase systems

Balanced Y-Δ Connection



$$V_{an} = V_p\angle 0^o$$

$$V_{bn} = V_p\angle -120^o$$

$$V_{cn} = V_p\angle 120^o$$

$$V_{ab} = \sqrt{3}V_p\angle 30^o = V_{AB}$$

$$I_{AB} = \frac{V_{AB}}{Z_\Delta}$$

$$V_{bc} = \sqrt{3}V_p\angle -90^o = V_{BC}$$

$$I_{BC} = \frac{V_{BC}}{Z_\Delta}$$

$$V_{ca} = \sqrt{3}V_p\angle 150^o = V_{CA}$$

$$I_{CA} = \frac{V_{CA}}{Z_\Delta}$$

Phase currents

Using KCL,

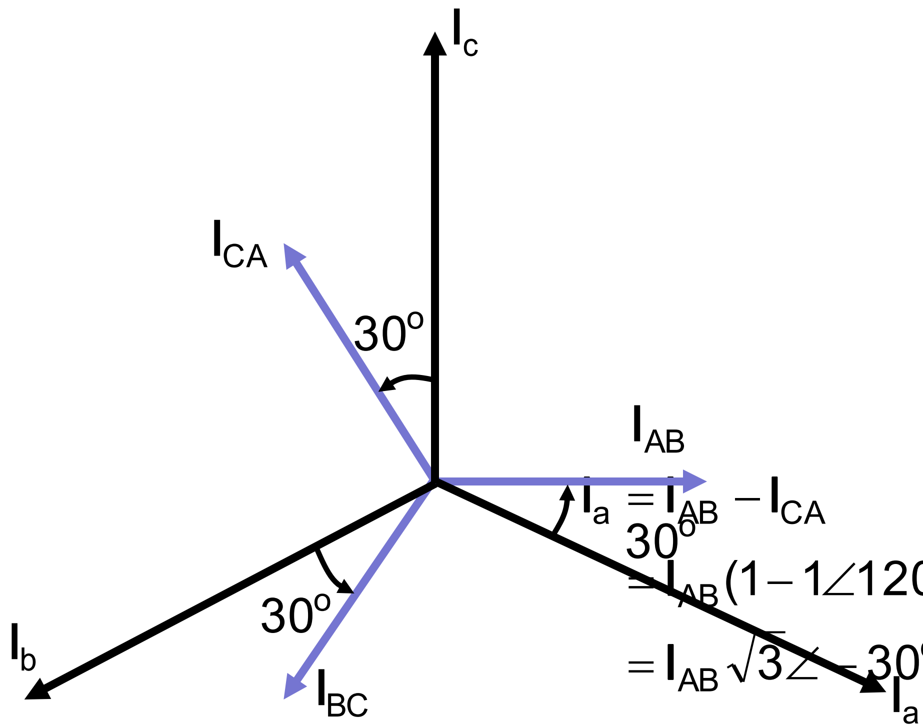
$$I_a = I_{AB} - I_{CA} = I_{AB}(1 - 1\angle 120^o) = I_{AB}\sqrt{3}\angle -30^o$$

$$I_b = I_{BC} - I_{AB} = I_{BC}(1 - 1\angle 120^o) = I_{BC}\sqrt{3}\angle -30^o$$

$$I_c = I_{CA}\sqrt{3}\angle -30^o$$

Balanced 3-phase systems

Balanced Y-Δ Connection



$$I_a = I_{AB} - I_{CA} = I_{AB}(1 - 1\angle 120^o) = I_{AB}\sqrt{3}\angle -30^o$$

$$I_b = I_{BC} - I_{AB} = I_{BC}(1 - 1\angle 120^o) = I_{BC}\sqrt{3}\angle -30^o$$

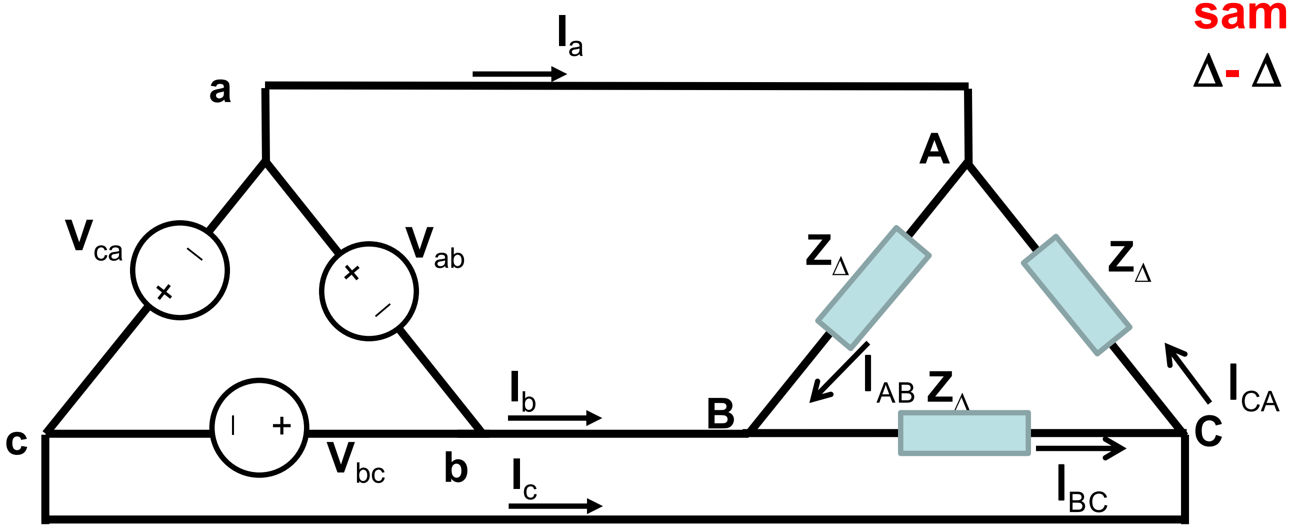
$$I_c = I_{CA}\sqrt{3}\angle -30^o$$

$$I_L = \sqrt{3} I_p$$

where $I_L = |I_a| = |I_b| = |I_c|$ and $I_p = |I_{AB}| = |I_{BC}| = |I_{CA}|$

Phase current LEADS line current by 30°

Balanced 3-phase Δ-Δ



Line-line voltage is the same as phase voltage in Δ- Δ

$$V_{ab} = V_p \angle 0^o$$

$$V_{bc} = V_p \angle -120^o$$

$$V_{cn} = V_p \angle 120^o$$

$$V_{ab} = V_{AB} \qquad I_{AB} = \frac{V_{AB}}{Z_\Delta}$$

$$V_{bc} = V_{BC} \qquad I_{BC} = \frac{V_{BC}}{Z_\Delta}$$

$$V_{ca} = V_{CA} \qquad I_{CA} = \frac{V_{CA}}{Z_\Delta}$$

Phase currents

Using KCL,

$$I_a = I_{AB} - I_{CA}$$
$$= I_{AB}(1 - 1\angle 120^o)$$
$$= I_{AB}\sqrt{3}\angle -30^o$$

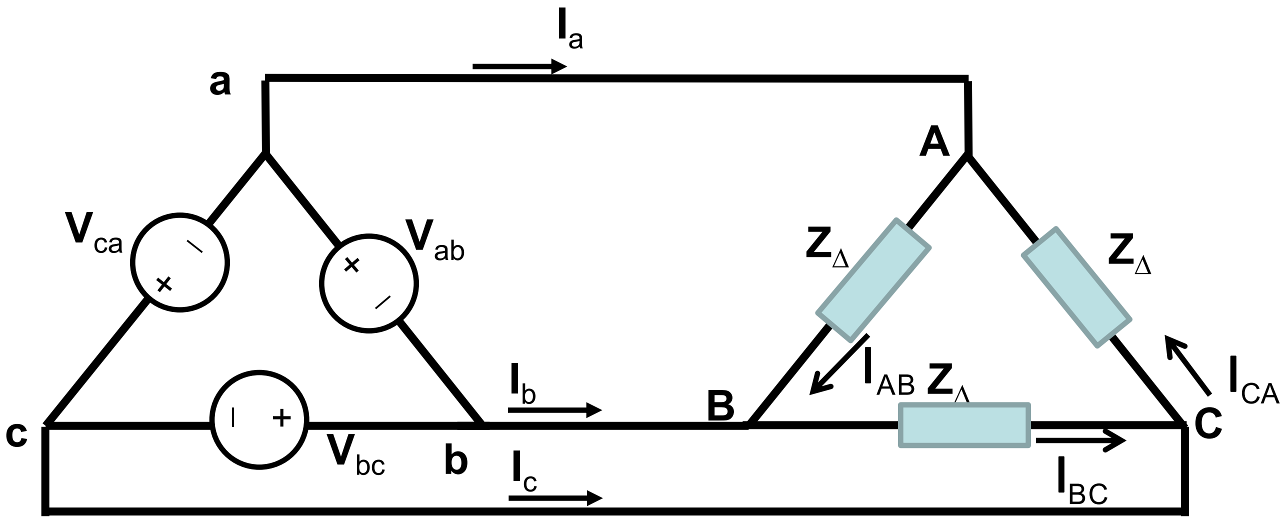
$$I_b = I_{BC} - I_{AB}$$
$$= I_{BC}(1 - 1\angle 120^o)$$
$$= I_{BC}\sqrt{3}\angle -30^o$$

$$I_c = I_{CA}\sqrt{3}\angle -30^o$$

line currents

Balanced 3-phase systems

Balanced Δ-Δ Connection



$$V_{ab} = V_p \angle 0^o$$

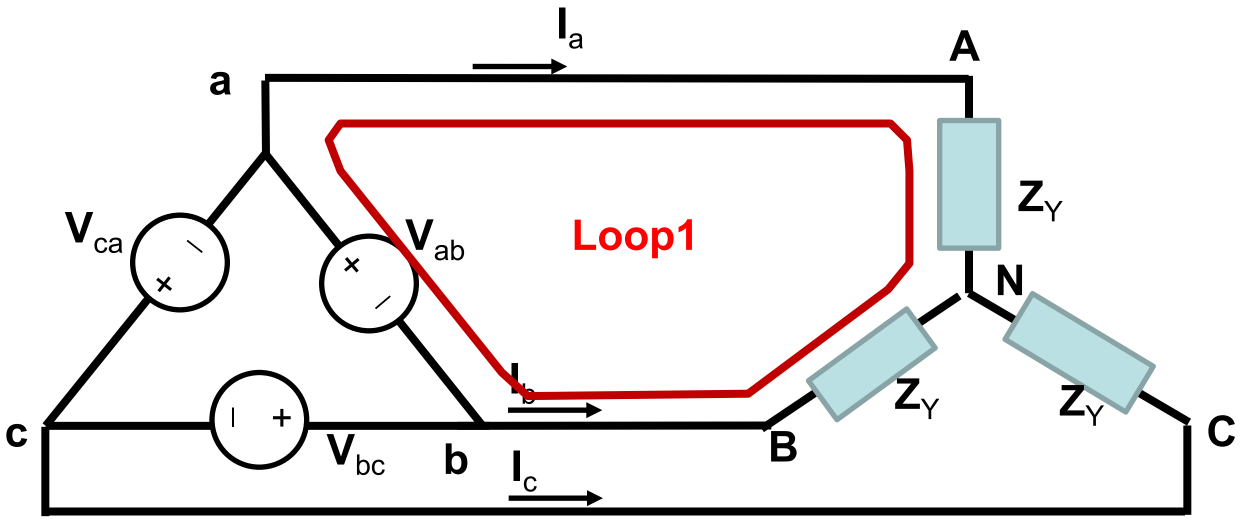
$$V_{bc} = V_p \angle -120^o$$

$$V_{cn} = V_p \angle 120^o$$

Alternatively, by transforming the Δ connections to the equivalent Y connections per phase equivalent circuit analysis can be performed.

Balanced 3-phase systems

Balanced Δ-Y Connection



$$\mathbf{V}_{ab} = V_p\angle 0^o$$

$$\mathbf{V}_{bc} = V_p\angle -120^o$$

$$\mathbf{V}_{ca} = V_p\angle 120^o$$

How to find $\mathbf{I}_a$?

Loop1 $-\mathbf{V}_{ab} + Z_Y\mathbf{I}_a - Z_Y\mathbf{I}_b = 0 \qquad \Rightarrow \mathbf{I}_a - \mathbf{I}_b = \frac{\mathbf{V}_{ab}}{Z_Y}$

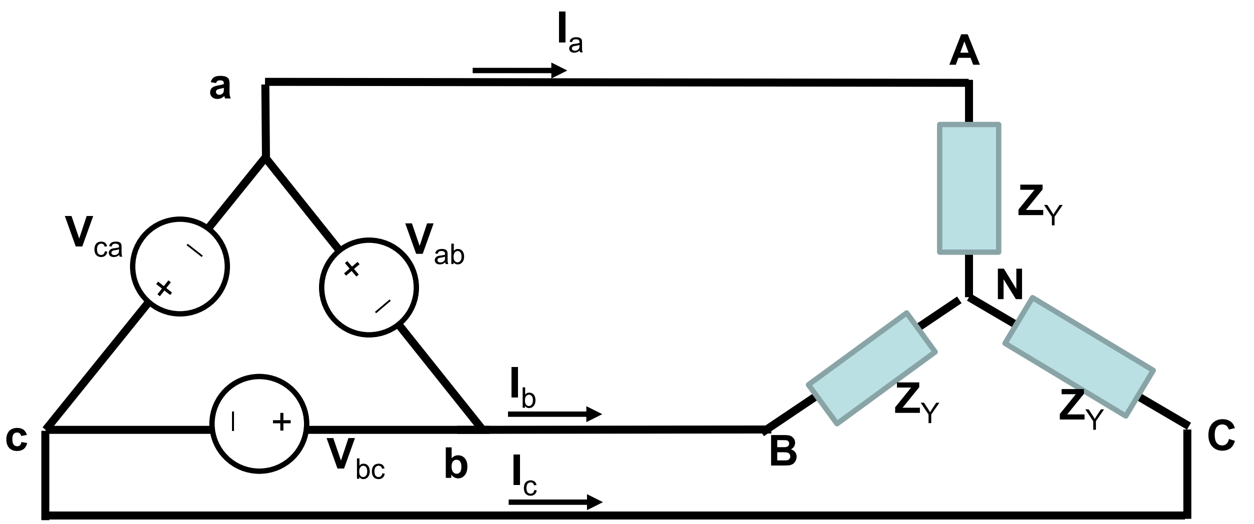
Since circuit is balanced, $\mathbf{I}_b = \mathbf{I}_a\angle -120^o$ $\quad \Rightarrow \mathbf{I}_a - \mathbf{I}_b = \mathbf{I}_a(1 - 1\angle(-120^o))$

$$= \mathbf{I}_a\sqrt{3}\angle 30^o$$

Therefore $\quad \mathbf{I}_a = \frac{V_p/\sqrt{3}}{Z_Y}\angle -30^o$

Balanced 3-phase systems

Balanced Δ-Y Connection



$$V_{ab} = V_p \angle 0^o$$

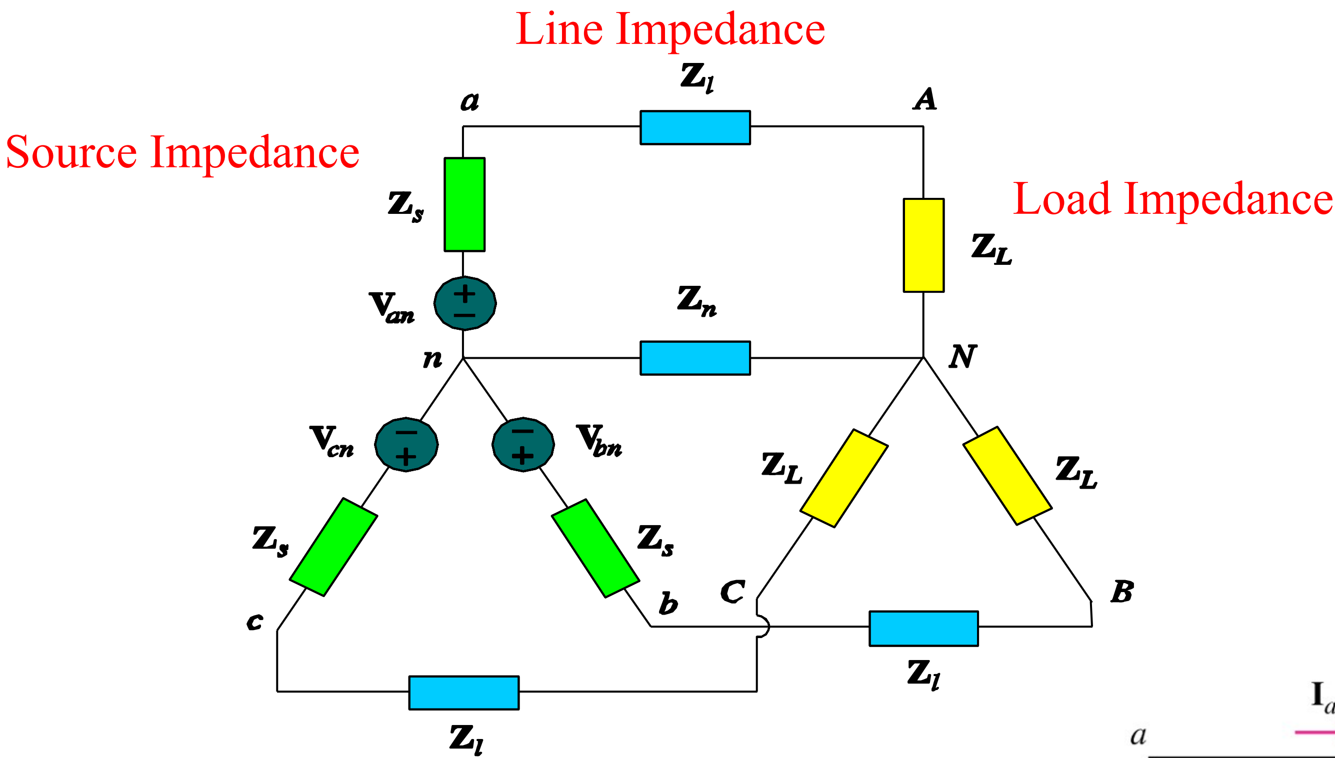
$$V_{bc} = V_p \angle -120^o$$

$$V_{ca} = V_p \angle 120^o$$

How to find $\mathbf{I}_a$? (Alternative)

Transform the delta source connection to an equivalent Y and then perform the per phase circuit analysis

- A balanced Y-Y system, showing the source, line and load impedances.



Equivalent Circuit